SHARAD Workshop – 45th LPSC

– FINDING DIELECTRIC PROPERTIES –

Daniel C. Nunes, JPL
Interaction of SHARAD happens in diverse ways:

- ionosphere - conductivity
- at the surface and other interfaces – contrast in permittivity, roughness
- through a given subsurface volume – conductivity, volume scattering

Here, I examine in this presentation a simplified case:

- smooth surface with a given dielectric contrast
- signal loss due to conductivity through a given volume

Disregarded in this presentation are the effects arising from:

- ionosphere
- surface roughness
**Basic Principles and Vocabulary:**

**Permittivity** – describes how charge migration and dipole re-orientation occurs in a medium submitted to an electric field.

\[
\varepsilon_0 = 8.85 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}
\]

**Relative Permittivity** – the ratio between the permittivity of a material and that of vacuum

\[
\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}
\]

(or 1 for vacuum/free-space)

**Dielectric Constant** – another name to relative permittivity

**Conductivity** – describes how easily the flow of charge (current) occurs in a medium submitted to an electric field.

\[
\sigma = \frac{J}{E}
\]

(current density/electric field).
Complex permittivity – \( \varepsilon = \varepsilon' + i\varepsilon'' \)

- the real portion relates to the dielectric constant, which we have already seen
- the imaginary portion relates to the conductivity...

\[
\varepsilon'' = \frac{\sigma}{\omega}, \quad \omega = 2\pi f
\]

Loss tangent – the ratio between the real and imaginary portions of the complex permittivity and a measure of the lossiness of a material (the lossier a material is, the more attenuated a signal becomes as it travels through the medium).

\[
\tan \delta = \frac{\varepsilon''}{\varepsilon'}
\]
Incident power \( P_0 \)

Power of surface reflection \( P_s \)

Power of sub-surface reflection \( P_{ss} \)

\( \varepsilon_0 \) \( t_s \) \( t_{ss} \) \( \varepsilon_1 \) \( h_1 \) \( \varepsilon_2 \)
Delay between surface and subsurface reflections

\[ \Delta t = t_{ss} - t_s \]
Propagation I

Delay between surface and subsurface reflections

\[ \Delta t = t_{ss} - t_s \]

But this delay is due to the signal travel time

\[ \Delta t = \frac{2h_1}{v} \]
Delay between surface and subsurface reflections

\[ \Delta t = t_{ss} - t_s \]

But this delay is due to the signal travel time

\[ \Delta t = \frac{2h_1}{v} \]

In dielectric media, the propagation velocity “v” depends on the dielectric constant

\[ v = \frac{c}{\sqrt{\varepsilon'_1}} \]
Delay between surface and subsurface reflections

\[ \Delta t = t_{ss} - t_s \]

But this delay is due to the signal travel time

\[ \Delta t = \frac{2h_1}{v} \]

In dielectric media, the propagation velocity “\(v\)” depends on the dielectric constant

\[ \nu = \frac{c}{\sqrt\varepsilon_1} \quad \text{c is speed of light in vacuum} \]

If \(h_1\) is known and \(\Delta t\) measured, then the last two equations can be combined and rearranged into

\[ \varepsilon_1 = \left( \frac{c\Delta t}{2h_1} \right)^2 \]
Finding Dielectric Constant – Case 1: NPLD
Need to define the domain to which the dielectric constant (>1) will be applied.
Finding Dielectric Constant – Case 1: NPLD
A dielectric constant of 3 will cause the basal reflections to lie in the same level as the surrounding Vastitas Borealis. Nearly pure ice is a good solution for the NPLD.
Finding Dielectric Constant – Case 2: Pedestals
One may test different values for dielectric constant and test the behavior.

- water ice : $\varepsilon' = 3$
- most rocks : $4 \leq \varepsilon' \leq 9$
Finding Dielectric Constant – Case 2: Pedestals

- Measure vertical offset between depth of basal reflector and the surrounding terrain
- Zero offset gives the dielectric estimate for the material.
- In this case, multiple radargrams give different answers!
  - material is heterogeneous
  - interface roughness/topography add uncertainty to the offset
  - “basal” reflector is not so “basal”
  - uncertainties dominate in the case of thinner deposits
A reflection coefficient \((r)\) describes the amount of energy that is reflected off a dielectric interface.

\((1-r)\) gives the amount of energy transmitted into the next medium.

Normal incidence case:

\[
r_{01} = \frac{\sqrt{\varepsilon_0'} - \sqrt{\varepsilon_1'}}{\sqrt{\varepsilon_0'} + \sqrt{\varepsilon_1'}}
\]
In addition to transmission losses described by $r$, the signal is attenuated along its propagation path due to the loss-tangent:

$$P(h_1) = P_t \exp \left[ \frac{-4\pi h_1}{\lambda} \sqrt{\frac{\varepsilon_1'}{2} \left( \sqrt{1 + (\tan \delta)^2} - 1 \right)} \right]$$

The greater the frequency or shorter the wavelength, the greater the attenuation.
Total Losses = Transmission Losses + Path Losses
• Consider the case of a sloping subsurface interface, where the layer material is assumed to be homogeneous
  
  - transmission losses are constant throughout

• Thickness of layer changes laterally (h(x))
  
  - path losses change along the slope
  
  - greater path loss where layer is thicker

• If any changes are seen in \((P_{ss}/P_s)\) along slope, then \(\tan\delta_1\) can be derived
Fig. 5 - Portions of two SHARAD radargrams for in north central Amazonis Planitia (see Figures 2 and 6). North is toward the left; image width is about 315 km. Note that the subsurface reflecting horizon parallels the topography of the ridge and is continuous beneath this structure. From Campbell et al. (2008).

Fig. 6 - SHARAD subsurface reflections presented as color overlay for variations in round-trip echo delay on a portion of the geologic map by Tanaka et al. [2005]. From Campbell et al. (2008).

No thickness information from other data sets, therefore, no best-estimate for $\varepsilon'$ of layer. Need to assume the plausible range for geologic materials (3-9).
Fig. 7 - SHARAD subsurface reflector power loss (in dB) versus round-trip time delay for orbit tracks over Amazonis Planitia. Power values are normalized to the average of surface echo power along each track and fit with a simple power law (straight lines). See Table 1 for best fit slope values. From Campbell et al. (2008).
Interpretation

- There are many manuscripts and papers about dielectric properties of different materials.

- In general, H$_2$O and CO$_2$ ices have low permittivity and loss tangent, while silicates have higher permittivity and loss-tangent.

- Water, and especially salt water, have very high permittivities and loss-tangent. A putative water table would produce very strong reflections and attenuation.

- Mixtures and porosity also modulate the effective permittivity/loss-tangent.

  - Bulk CO$_2$ ice : $2.20 + 2.12 \times 10^{-6}$
  - Bulk H$_2$O ice : $3.15 + 6.30 \times 10^{-4}$
  - Basalt (low) : $5.4 + 1.0 \times 10^{-3}$
  - Shergottite : $8.8 + 1.7 \times 10^{-2}$
  - Altered basalt : $15 + 1.5$

**Fig. 6** - Color map showing the effective permittivity $\varepsilon^{\prime\prime}$mix of a mixture of water ice w/silicates or porous silicates obtained with the deLoor mixing model, with $\varepsilon^{\prime\prime}$ice = 3.15. From Nunes et al. (2011).
Interpretation

- Any other independent information about composition, porosity, geologic context helps addressing the non-uniqueness aspect of radar-derived permittivities.

References