Sounding Radars 101

Roger Phillips
Isaac Smith

VHF antenna
On Apollo 17
If you already know that

\[ \text{distance} = \text{velocity} \times \text{time} \]

you’re in great shape, though really

\[ 2 \times \text{distance} = \text{velocity} \times \text{time} \]
What is radar doing?

- Pulse of energy sent
- Some energy is reflected
- Intensity and time are recorded

Diagram: A radar dish emitting a pulse of energy, with reflected energy recorded over time.
Two Way Time (TWT)

Signal travels away from transmitter

Reflects off of a surface

Travels back to receiver

Step 1 time = \( \frac{h}{v} \)

Step 2 time = \( \frac{h}{v} \)

Total time = \( 2 \times \frac{h}{v} \)

\[
V = \frac{1}{\sqrt{\mu \varepsilon}} \approx \frac{1}{\sqrt{\mu_0 \varepsilon' \varepsilon_0}} = \frac{V_0}{\sqrt{\varepsilon'}}
\]
Velocities in Media

Signal Velocity depends on real part of permittivity, $\varepsilon'$

$$V \approx V_0 / \sqrt{\varepsilon'}$$

<table>
<thead>
<tr>
<th>Space</th>
<th>CO$_2$</th>
<th>H$_2$O</th>
<th>Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0 = \frac{3 \times 10^8}{\sqrt{1}}$</td>
<td>$V = \frac{3 \times 10^8}{\sqrt{2.1}}$</td>
<td>$V = \frac{3 \times 10^8}{\sqrt{3.15}}$</td>
<td>$V = \frac{3 \times 10^8}{\sqrt{\sim 4 \text{ to } 12}}$</td>
</tr>
</tbody>
</table>

$\varepsilon = \varepsilon' + i\varepsilon''$; $\tan \delta = \varepsilon'' / \varepsilon'$

Basic quest is for depth, but estimates of $\varepsilon'$ & $\tan \delta$ constrain composition and porosity
Signal return time

\[ t = \frac{d}{v} \]

\[ t_{\text{air}} = \frac{2h}{v_{\text{air}}} \]

\[ t_{\text{rock}} = \frac{2d}{v_{\text{rock}}} \]

Total time = \( t_{\text{air}} + t_{\text{rock}} \)

- Air permittivity: \( \varepsilon' \sim 1 \)
- Air speed: \( v_{\text{air}} = c \)
- Rock permittivity: \( \varepsilon' \sim 9 \)
- Rock speed: \( v_{\text{rock}} = \frac{c}{3} \)

\[ h = \text{huge} \]
\[ d = 10 \]

These are really relative permittivities; i.e., divided by \( \varepsilon_0 \)
Build a radargram

Stack individual echo traces along track to build up a **radargram**
Horizontal resolution

a) 

b) 

Height

H

d

e_r

Along Track

c) 

Surface signal

Subsurface signal

t_{ss} = 2d\sqrt{\frac{e_r}{C_0}}
Vertical resolution
Time-bandwidth product ~ unity;

$$\Delta t \Delta f \sim 1; \quad \Delta t \sim 1 / \Delta f; \quad \Delta h \approx \frac{V_0}{2\sqrt{\varepsilon'}} \frac{1}{\Delta f}$$

Transmitter signal is a chirp, which enhances output energy by spreading the bandwidth over time, $E = A^2 \times t; \quad A^2 = P$

There is a price to be paid for this.
Dreaded Sidelobes

- Chirp signal has sharp cutoffs in frequency domain
- Rectangle (box car) $\text{sin}(x)/x$ (sinc function)
Reflected signal not so simple

Sidelobes or ss reflectors or both? We have been fooled before!

Tradeoff in weighting to suppress sidelobes & resolution
Surface moves up
Subsurface moves down
due to decreased velocity
Again, surface moves up
Subsurface moves down
due to decreased velocity
time

media 1
$\varepsilon = 1$

media 2
$\varepsilon = 4$

media 3
$\varepsilon = 9$
$\varepsilon'$ 3.8

This + next 4 slides: time to depth w/ different $\varepsilon'$
$\varepsilon'$  3.4
\varepsilon' \quad 3.0
$\varepsilon'$  2.6
$\varepsilon'$  2.2
Depth conversion w/ multiple $\varepsilon'$
How deep is a reflector?
How deep is a reflector?

\[
\text{631 pixels} \times 37.5 \text{ ns per pixel} \Rightarrow 23.7 \mu \text{s TWT}
\]
How deep is a reflector?

23.7 μs TWT
11.8 μs OWT

\[ h = v \cdot OWT \]
\[ v = c / \sqrt{3.15} \]
\[ c = 3 \times 10^8 \text{ m/s} \]

\[ h = 3 \times 10^8 / \sqrt{3.15} \cdot 11.8 \mu s \]

\[ h = 2000 \text{ m} \]
Sometimes you get clutter nearby mountain (not nadir)
Sometimes you get clutter

Clutter

Source of clutter

Nadir track
Sometimes you get clutter
<table>
<thead>
<tr>
<th></th>
<th>MARSIS</th>
<th>SHARAD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency Bands</strong></td>
<td>1.3-2.3 MHz, 2.5-3.5 MHz, 3.5-4.5 MHz, 4.5-5.5 MHz</td>
<td>15-25 MHz</td>
</tr>
<tr>
<td><strong>Vertical Resolution</strong></td>
<td>~100 m (1 MHz BW)</td>
<td>~10 m (10 MHz BW)</td>
</tr>
<tr>
<td>(ε’ = 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Penetration Depth</strong></td>
<td>&gt; 3 km in ice-dominated material</td>
<td>Few 100 m in rock</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Up to 2 km in ice</td>
</tr>
<tr>
<td><strong>Horizontal Resolution</strong></td>
<td>5-9 km x 15-30 km</td>
<td>0.3-1 km x 3-6 km</td>
</tr>
<tr>
<td>(along-track x cross-track)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Processing</strong></td>
<td>Mostly on-board</td>
<td>Mostly on the ground</td>
</tr>
</tbody>
</table>
Congratulations!
You have passed
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