JOHNS HOPKINS
UNIVERSITY
Applied Physics Laboratory
Laurel MD 20723-6099

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To: Distribution

From: J R Jensen

Subject: Forerunner transmit polarization ellipse as observed at Green Bank Radio Telescope on February 28 and March 1, 2009

References: [1] "Computing the orientation of the Mini-RF principal axes relative to the Green Bank antenna", F. S. Turner, JHU/APL memo SIS-09-019, October 20, 2009.

[2] "LRO Mini-RF S-band transmit polarization ellipse as observed at Green Bank on August 23, 2009", J. R. Jensen, JHU/APL memo SRO-09M-15, August 24, 2009.

[3] "XY and YX definitions", email from Frank Ghigo (GBT), October 8, 2009.

[4] "Calibration Experiment with Arecibo Radio Telescope", H. B. Sequeira, F. S. Turner, and L. Nguyen, JHU/APL memo SER-09-010, March 5, 2009.

[5] "Forerunner gain and phase balance as observed during nadir calibration on January 18, 2009", J. R. Jensen, JHU/APL memo SRO-09M-19, October 19, 2009.

[6] "Derivation of Elliptically-polarized transmit/linearly-polarized receive Hybrid Polarity Parameters with Initial Applications", M. R. Keller, JHU/APL memo SRO-09M-05, March 12, 2009.

Introduction

Two scans of the Forerunner Mini-RF antenna at S-band were conducted with the Green Bank Radio Telescope on February 28 and March 1, 2009. An "azimuth" scan was performed on February 28. An "elevation" scan was performed on March 1. A rotation of the Chandrayaan-1 spacecraft of 147.8° was performed between these two scans (Reference [1]).

This report describes the analysis of the data collected at Green Bank during these two scans and discusses the results in the context of independent measurements of the receiver performance observed during an uplink from the Arecibo Radio Telescope and radar data collected when Forerunner was in a nadir-looking orientation at the Moon.

Coordinate Systems

Figure 1 shows the coordinate systems that will be used throughout this report. Figure 1 shows the Forerunner and Green Bank coordinates as viewed by an observer that is behind the Green Bank antenna and is looking through the Green Bank antenna at the front (radiating) surface of the Forerunner antenna. The Forerunner beam direction and the Green Bank +Z-axis are parallel and are out-of-the-page in the figure. The direction of circulation for the Arecibo uplink test and the field transmitted from Forerunner are also shown on this figure for use later.

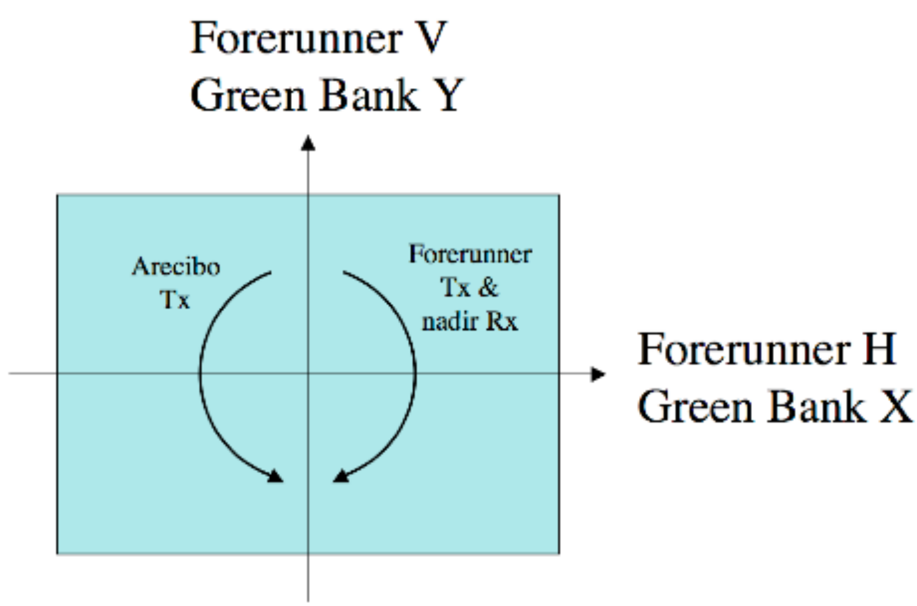

Figure 1. Green Bank and Forerunner coordinate systems. The Forerunner beam is out-of-the-page ($H \times V$), as is the Green Bank +Z-axis ($X \times Y$).

Geometries of the Azimuth and Elevation Scans

After projection of the Forerunner axes onto the Green Bank antenna, the angle between the Forerunner H-axis and the Green Bank X-axis is –103.3˚ for the azimuth scan. The corresponding angle for the elevation scan is 108.9˚. These two projections are shown in Figures 2 and 3. The rotation between these two scans is 212.2˚, or –147.8˚. This requires a more complicated analysis than what is described in Reference [2] for a simple ±90˚ rotation. The calculation of the geometries is described in Reference [1].

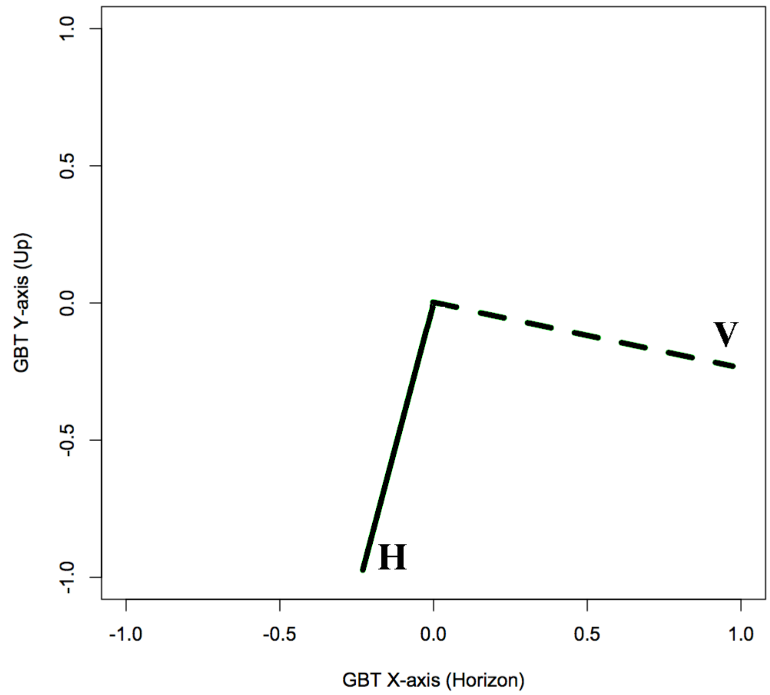

Figure 2. Projection of the Forerunner coordinates onto the Green Bank antenna for the azimuth scan. The angle between the Forerunner H-axis and the Green Bank X-axis is –103.3˚.

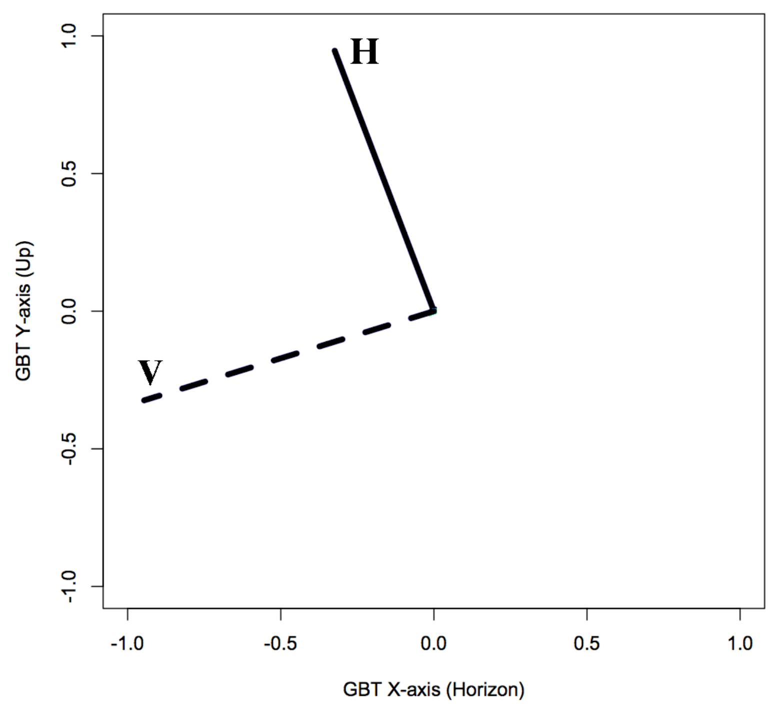

Figure 3. Projection of the Forerunner coordinates onto the Green Bank antenna for the elevation scan. The angle between the Forerunner H-axis and the Green Bank X-axis is 108.9˚.

<u>Green Bank Observations</u>

The plots of the Green Bank data are contained in Figures 4 and 5. "XX" and "YY" are the powers in the X and Y linear receiver channels, whose alignment is as shown in Figure 1. "XY" and "YX" are the real and imaginary parts of the cross-term (product of X and Y*). The values of these quantities at the center of the scan are taken from these plots and are listed in Table 1. Because the peaks of the "YX" plots do not occur at the same time for which the "XX" and "YY" plots have their maximum, the values in Table 1 are lower than the peak values in the plots. In order to be meaningful, the values must all pertain to a single instant of time.

The problem addressed here is to determine a set of receiver gain coefficients and incident field components that fit the data in Table 1 and for which the field components are rotated between the two scans as shown in Figures 2 and 3. The determination of these gains and field components involves solving a quadratic equation, which has two solutions. The two solutions correspond to the opposite polarization of the incident field. That is, one solution is left-hand circularly polarized and the other is right-hand.

Green Bank's +Z-axis points from space to the antenna. The field transmitted from Forerunner is left-hand circularly polarized, in the forward sense, as observed by the APL ground station. This means that the phase of the Y-component of the field will lead the phase of the X-component and $\arg\left\{F_X F_Y^*\right\} < 0$ for the correct solution. This understanding was confirmed by Frank Ghigo at Green Bank (Reference [3]). This condition has been used to choose between the two possible solutions.

Table 1. Measured values of the power for each receiver channel ("XX" and "YY") and the real and imaginary components of the cross-term ("XY" and "YX"), for the Forerunner Green Bank measurements.

	azimuth scan	elevation scan
XX	6.70×10^7	6.57×10^7
YY	7.23×10^7	7.32×10^7
XY	6.95×10^7	6.72×10^7
YX	0.37×10^7	-1.72×10^7

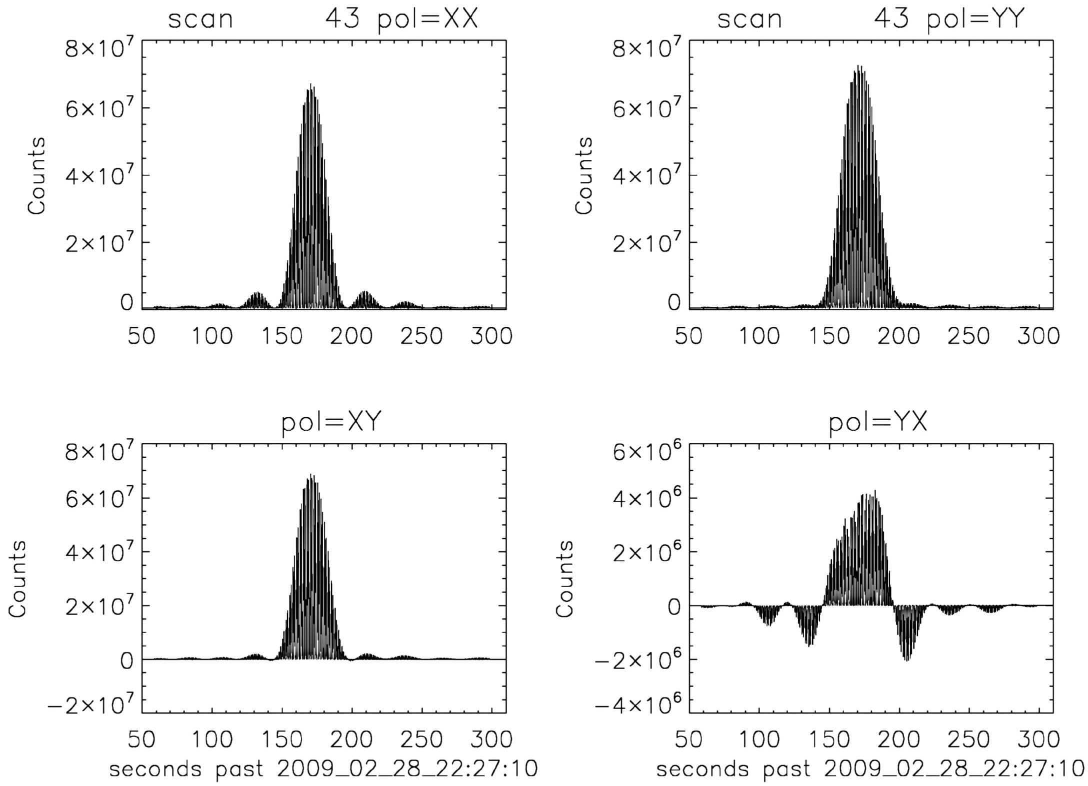

Figure 4. Receiver channel power and cross-terms for the azimuth scan.

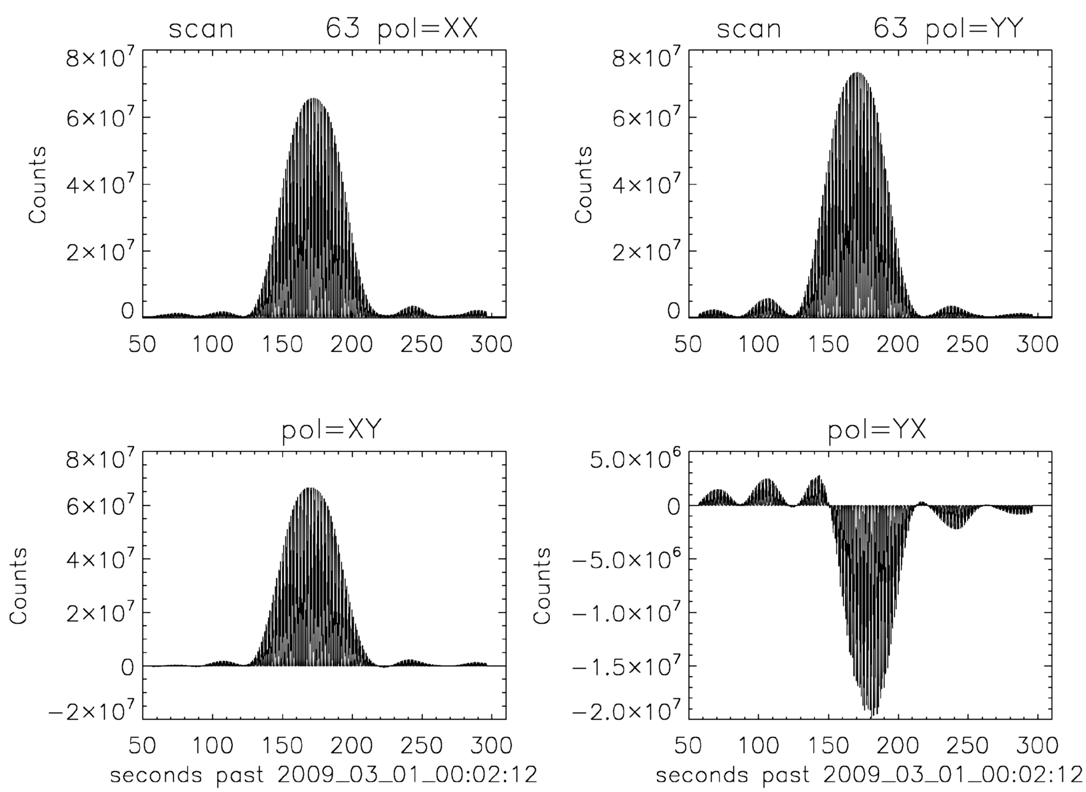

Figure 5. Receiver channel power and cross-terms for the elevation scan.

Solution for the gains and field components

Arbitrarily setting the gain of the "X" receiver channel to 1 and specifying that the field component on the X-axis be real for the azimuth scan (the analysis leaves some things undetermined, but also unimportant), the values in Table 1 can be broken down as follows.

Receiver gain terms

$$A_{\mathrm{X}} = 1.$$
$$A_{\mathrm{Y}} = 0.0913 - 0.8203i$$

Field components for azimuth scan

$$F_{\mathrm{X}} = 8185.3$$
$$F_{\mathrm{Y}} = 1682.8 + 10164.1i$$

Field components for elevation scan

$$G_{\mathrm{X}} = -6030.0 + 5416.5i$$
$$G_{\mathrm{Y}} = -5786.1 - 8601.3i$$

Demonstration of the solution

The field components from the elevation scan are related to the components of the azimuth scan by a rotation angle of –147.8°. That is,

$$\begin{pmatrix} G_{\mathrm{X}} \\ G_{\mathrm{Y}} \end{pmatrix} = \alpha \begin{pmatrix} \cos(-147.8^\circ) & -\sin(-147.8^\circ) \\ \sin(-147.8^\circ) & \cos(-147.8^\circ) \end{pmatrix} \begin{pmatrix} F_{\mathrm{X}} \\ F_{\mathrm{Y}} \end{pmatrix}$$

where α is an overall real scale factor for the second scan that is equal to 1.0001 for this case. Such a scale factor would account for a difference in the atmospheric absorption, for example.

Using these values, the computed powers for the azimuth scan are

$$XX = |A_{\mathrm{X}} F_{\mathrm{X}}|^2 = 6.70 \times 10^7$$
$$YY = |A_{\mathrm{Y}} F_{\mathrm{Y}}|^2 = 7.23 \times 10^7$$
$$XY + YXi = A_{\mathrm{X}} F_{\mathrm{X}} A_{\mathrm{Y}}^* F_{\mathrm{Y}}^* = 6.95 \times 10^7 + 0.37 \times 10^7 i$$

and for the elevation scan

$$XX = |A_X G_X|^2 = 6.57 \times 10^7$$
$$YY = |A_Y G_Y|^2 = 7.32 \times 10^7$$
$$XY + YXi = A_X G_X A_Y^* G_Y^* = 6.72 \times 10^7 - 1.72 \times 10^7 i$$

These agree with the observed values listed in Table 1 taken from the curves in Figures 4 and 5.

The critical assumption in the above analysis is that the Green Bank receiver operated in an identical fashion for both of the scans. The only changes that are assumed between the two scans are a rotation and an overall scale factor applied to the incident field. A comparison of the results to independent measurements below encourages acceptance of this assumption.

Projection of the Fields onto Forerunner Antenna

The above results for F_X and F_Y pertain to the geometry shown in Figure 2. In order to determine the field components in the Forerunner coordinates, the fields must be rotated by 103.3° to align the Green Bank axes and the Forerunner axes as shown in Figure 1.

The transformation is

$$\begin{pmatrix} F_H \\ F_V \end{pmatrix} = \begin{pmatrix} \cos(103.3^\circ) & -\sin(103.3^\circ) \\ \sin(103.3^\circ) & \cos(103.3^\circ) \end{pmatrix} \begin{pmatrix} F_X \\ F_Y \end{pmatrix}$$

The result of the rotation is

Field components rotated into Forerunner frame

$$F_H = -3520.7 - 9891.5i$$
$$F_V = 7578.7 - 2338.2i$$
$$\frac{|F_V|}{|F_H|} = 0.755 = -2.44 \text{ dB}$$
$$\arg\{F_V F_H^*\} = 92.4^\circ$$
$$\text{axial ratio} = 2.46 \text{ dB}$$

This is the final result for the calculation of the Forerunner transmitted field.

Discussion

The above results can be compared to other, independent measurements of the Forerunner instrument. In particular, an uplink test with the Arecibo Radio Telescope was conducted on January 30, 2009 and a nadir collection was performed on January 18, 2009. The uplink test characterizes the complete receiver components of the radar, while the Green Bank test addressed above characterizes the complete transmit components. The nadir test is an end-to-end test of the system. A full understanding of the instrument performance needs to integrate all of these observations.

Gain and Phase Balance Observations from Arecibo Uplink

Analysis of the results from the Arecibo uplink test are presented in Reference [4]. From that test, it was determined that $|V|/|H| = 0.3 \pm 0.25$ dB, where $|V|$ is the amplitude of the vertical channel signal and $|H|$ is the amplitude of the horizontal channel signal. This is a gain imbalance due to the antenna and the receiver system. The digital data are well positioned within the available dynamic range for this measurement.

For the Arecibo measurements, the electric fields are not directly observed but are indicated by the digital output of the Forerunner receiver. The relationship between the relative phase of H and V components of the radiated field at the antenna surface and at the digital output is

$$\Delta\phi_{\mathrm{out,V-H}} = -\left[\Delta\phi_{\mathrm{ant,V-H}} + \Delta\phi_{\mathrm{RX,V-H}}\right]$$

where $\Delta\phi_{\mathrm{out,V-H}}$ is the relative phase in the digital output, $\Delta\phi_{\mathrm{ant,V-H}}$ is the relative phase at the antenna surface, and $\Delta\phi_{\mathrm{RX,V-H}}$ is the contribution to the relative phase due to the complete receiver system. The minus sign in the above equation accounts for a spectral inversion that takes place within the Forerunner receiver due to a high-side mixer and this reverses the sense of the phase difference. Conceptually, writing the phase difference in this way corresponds to the receiver phase being applied prior to the phase inversion.

For the Arecibo uplink test, $\Delta\phi_{\mathrm{ant,V-H}} = -90^\circ$ (LHC in the forward sense). The measurement result is $\Delta\phi_{\mathrm{out,V-H}} = 46.5^\circ \pm 1.5^\circ$ for this test. (Reference [4]). Therefore, $\Delta\phi_{\mathrm{RX,V-H}} = 43.5^\circ \pm 1.5^\circ$. This represents the receiver contribution to the observed phase of the digital data.

End-to-end Prediction

Given the Arecibo and the Green Bank measurements, the prediction of the end-to-end power balance is $\left(|V|/|H|\right)_{\mathrm{E-E}} = 0.3 \pm 0.25 - 2.44 = -2.14 \pm 0.25$ dB. The predicted phase balance is a $\Delta\phi_{\mathrm{out,V-H}} = -\left[92.4^\circ + 43.5^\circ \pm 1.5^\circ\right] = -135.9^\circ \pm 1.5^\circ$. It is assumed in making these predictions that the nadir scattering is completely specular. That is, it is assumed that the power received on the horizontal channel is proportional to the power that is transmitted on the horizontal channel. Similarly for the vertical channel. Diffuse scattering will bring the power ratio closer to unity, but should not introduce a phase bias.

Nadir Observations

The available nadir data suffer from two imperfections. First, the digital data are clipped with most pulses having several samples that are at the extreme values of +127 and –128. Second, the range gate window captures does not capture the entire pulse. Only the leading portion of the pulse is captured. These data are discussed in Reference [5].

When only the pulses from the nadir collection that do not exhibit clipping or severe truncation (truncation that leaves fewer than 50 samples) are considered, the observed power ratio is $\left(|V|/|H|\right)_{\mathrm{E-E}} = -2.55 \pm 0.25$ dB. The corresponding phase balance is $\Delta\phi_{\mathrm{out,V-H}} = -138^{\circ} \pm 2.7^{\circ}$. These observations are shown in Table 2, along with the end-to-end predictions.

Table 2. End-to-end prediction and nadir calibration observations for the channel-to-channel gain and phase balances

	end-to-end prediction	nadir observation
gain balance (V/H)	–2.14±0.25 dB	–2.55±0.25 dB
phase balance (V-H	–135.9˚±1.5˚	–138˚±2.7˚

The power balance is lower than the end–to-end prediction by 0.4 dB, but both have an uncertainty of 0.25 dB. The phase balance is different from the end-to-end prediction by 2.9˚ but again they are within the combined uncertainties of the prediction and the observation. These observations provide confirmation of the analysis of the Green Bank data shown in Figures 4 and 5. The results are much different when the entire nadir data set is considered, which includes the clipped pulses, but the clipped pulses do not indicated the true balances of the radar system.

Summary

The result of the Green Bank data analysis is a Forerunner transmit polarization of

$$|F_{\mathrm{V}}|/|F_{\mathrm{H}}| = -2.44 \text{ dB}$$
$$\Delta\phi_{\mathrm{TX,V\text{-}H}} = \arg\left\{\mathrm{F_V} F_{\mathrm{H}}^{*}\right\} = 92.4^{\circ}$$
$$\text{axial ratio} = 2.46 \text{ dB}$$

where the sense of the phase difference is as shown in Figure 1. The phase of the field along the Forerunner vertical axis is 92.4˚ ahead of the phase of the field along the horizontal axis.

The results of the Arecibo uplink test are a receiver chain gain and phase imbalance of

$$|A_V|/|A_H| = 0.3 \pm 0.25 \text{ dB}$$
$$\Delta\phi_{RX,V\text{-}H} = \arg\{A_V A_H^*\} = 43.5^\circ \pm 1.5^\circ$$

These factors include the entire receiver chain of the antenna, cables, the analog receiver, and the digital elements.

Care must be exercised in applying these phase imbalances because of the phase-sense inversion within the receiver. As written, both phase terms are implicitly applied before the phase inversion. The end-to-end phase difference in the digital output is defined by

$$\Delta\phi_{E\text{-}E,V\text{-}H} = \Delta\phi_{out,V\text{-}H} = -\left[\Delta\phi_{TX,V\text{-}H} + \Delta\phi_{RX,V\text{-}H}\right]$$

and so the V-H phase difference in the digital data is predicted to be $-135.9^\circ \pm 1.5^\circ$ combining the Green Bank and Arecibo calibration results. The end-to-end measurement from the nadir collection shows good agreement with the predictions that are based on the separate Arecibo uplink and Green Bank downlink tests.

Because no further data will be collected with the Forerunner radar, these results are expected to be the final calibration.

The transmitted field from Forerunner is not circular, but is elliptical. The axial ratio of 2.46 dB corresponds to a field in which the LHC-to-RHC ratio of 17 dB. This is a significant enough ratio that the RHC contribution cannot be ignored in the polarimetric analysis of the Forerunner SAR images. The analysis contained in Reference [6] should be used to account for the contributions of the elliptically polarized field to the SAR images.

Notes

The values shown in Table 1 were taken directly from the plots shown in Figures 3 and 4. In doing so, a constraint was imposed to insure that

$$XX \times YY = XY^2 + YX^2 \text{ or } |A_X F_X|^2 |A_Y F_Y|^2 = |A_X F_X A_Y^* F_Y^*|^2$$

Because of the above constraint, the data in Figures 4 and 5 represent six constraints on the solution. There are two degrees of freedom in the solution for A_Y (A_X is set to 1), three degrees of freedom in the solution of F_X and F_Y (F_X is chosen to be real), and one degree of freedom in the solution of α. The number of constraints matches the number of degrees of freedom in the solution. The solution is not over-determined or under-determined.

As noted earlier in this memo, the “XY” and “YX” values taken from Figures 4 and 5 are not those of the peaks for those curves. The values in Table 1 correspond to the “XY” and “YX” plots at the time for which the “XX” and “YY” plots have their peak values. Those times correspond to the beam center.

No Faraday rotation has been included in the analysis presented here. The elevation angles for the Green Bank antenna for the first scan were between 51˚ and 53˚. For the second scan they were between 34˚ and 36˚. These angles are large enough that Faraday rotation should not be larger than about 1˚, based on some analysis that was done for the LRO scans performed on August 23, 2009. In that analysis, it was found that a very conservative estimate of the Faraday rotation for an elevation angle of 40˚ was 1.4˚. This would have a small impact on the analysis presented here.

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