Timing of Science Data for the GRAIL mission (JPL D-75620)

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Abstract

Science data from the Gravity Recovery and Interior Laboratory (GRAIL) mission are time tagged by clocks onboard the two GRAIL spacecraft. However, most of our scientific computer programs process data with Barycentric Dynamical Time (TDB) time tags. This document describes the timing of the science data and conversion of the onboard clock time tags to TDB scale. The GRAIL timing system has unique timing aspects that differ significantly from its predecessor mission, the Gravity Recovery and Climate Experiment (GRACE), which relied solely on the Global Positioning System (GPS) system for timing. For GRAIL, the timing data from the Deep Space Network (DSN), the onboard Time Transfer System (TTS) and frequency observations from the Radio Science Receiver (RSR) are combined to estimate the time tag conversion for the GRAIL science data. In order to assess the Lunar Gravity Ranging System (LGRS) measurement error due to GRAIL timing data sources, a second order derivation of the LGRS measurement model for the Dual One Way Range (DOWR) is presented. Furthermore, a derivation is presented for the extraction of the inter-satellite timing offset from the TTS ranging measurements. Finally an algorithm is presented to compute the time correlation between the LGRS clock and TDB time scale.

Contents

List of Figures

List of Tables

1 Introduction

The objective of the GRAIL mission is to determine with high accuracy the lunar gravity field for scientific research. The input data for the gravity field determination process are Ka phase measurements between the two GRAIL spacecraft which are used to compute the DOWR measurement. The DOWR measurement is then converted to instantaneous range, range rate and range acceleration measurements, which serve as inputs to the gravity field estimation process. Very accurate timing of the GRAIL measurements are crucial to achieve the high accuracy measurements needed for a high quality gravity field.

In this memorandum the GRAIL timing system and its performance will be discussed in terms of errors in the LGRS measurements. For this purpose we derived the LGRS model with second order terms and included the timing effects due to general relativity. The GRAIL mission can not use the GPS system for timing like the GRACE mission [6] did, but has to rely on the combination of several measurements to achieve the desired timing accuracy. Furthermore, we will asses how the design of the GRAIL timing system and its performance will impact the errors in the LGRS measurements. Based on this assessment, algorithms and processing choices will be presented and developed to insure that the required timing precision is achieved for the GRAIL mission.

2 Description of the GRAIL timing system

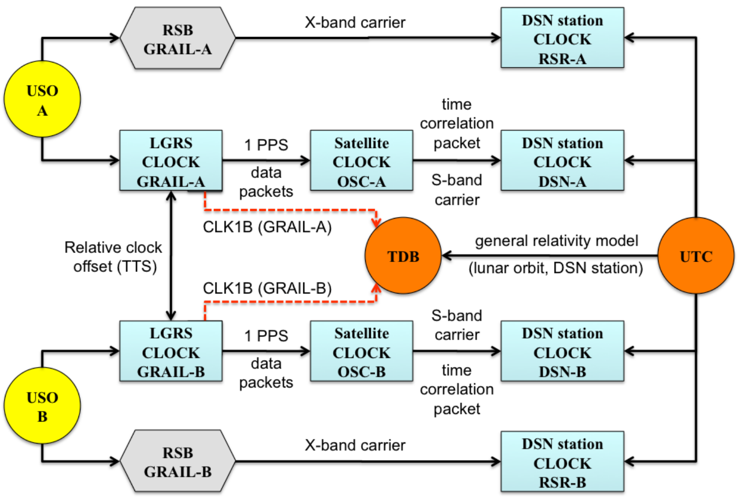

Figure 1: GRAIL clocks, models and measurements used for timing

This section describes the clocks and timing systems used by the GRAIL mission. The GRAIL mission uses the following three timing systems: 1) The LGRS clock, which is driven by an USO to create a highly stable clock, 2) the Onboard Spacecraft Clock (OSC), which is used in the timing correlation between the LGRS/OSC clocks and the time correlation between the OSC/DSN clocks and 3) the DSN clock, which is synchronized with highly stable time standard UTC. In figure 1 all the clocks used for GRAIL timing are shown with information or data flows between the different clocks and measurement systems. The ultimate goal is to provide a time correlation between the LGRS clocks and the TDB time scale, which will be provided in the Level1 product CLK1B.

2.1 LGRS clock

Each spacecraft has an LGRS clock which is driven by an USO to create a very stable clock. The LGRS clock is used for timing of the LGRS Ka phase measurement and the ranging data from the TTS. The LGRS clock has no notion of absolute time. Instead, the LGRS clock reading is with respect to the clock startup epoch. The required stability of the USO is listed in Table 1. The stability is given in terms of the Allan Deviation [1] which is the frequency change divided by the reference frequency ($\delta f/f$) averaged over a specific time interval.

Table 1: Allan Deviation requirements for GRAIL USO

Integration Time (seconds)	Allan Deviation
1	3×10^{-13}
10	3×10^{-13}
100	3×10^{-13}
1000	6×10^{-13}
57600	7×10^{-11}

2.2 Onboard spacecraft clock

The OSC is run by a crystal oscillator and has inferior stability characteristics compared to the LGRS USO. The OSC is used for time tagging all spacecraft data and the arrival of the LGRS data packets (which includes the LGRS 1 Pulse Per Second (PPS) packet) by the onboard computer. By time tagging the arrival of LGRS data packets and the arrival of the LGRS 1 PPS, a time correlation can be generated between the LGRS clock and the OSC. Furthermore, the OSC is used to time stamp spacecraft time correlation packets which are transmitted to a DSN station where the arrival time is recorded in UTC and thus providing a time correlation between the OSC and UTC. By combining the LGRS/OSC and OSC/UTC time correlation products, a time correlation between the LGRS and UTC can be determined and the OSC clock drops out. Hence the stability characteristics of the OSC do not affect the LGRS and UTC time correlation because the OSC error over short intervals (< 1 second) are < 1 micro second.

2.3 UTC clock used by DSN

The DSN uses very stable clocks which are maintained by the DSN Frequency and Timing Subsystem (FTS) [7]. The DSN time stamps the arrival of telemetry and radio metric tracking data in the UTC time scale, which is tied to the International Atomic Time (TAI) time scale. Based on FTS reports the real-time timing performance is at the 10^{-6} second level and post processing analysis improves the performance to the 10^{-9} second level.

3 LGRS DOWR measurement errors due to timing errors

3.1 LGRS DOWR measurement model

This subsection describes the mathematical model for the LGRS measurement in the presence of time tag errors of the phase measurement, phase errors due to the instrument, general relativity and random phase errors. Two mathematical descriptions are considered in this section, both were developed for the GRACE mission [4],[8]. We re-derived the mathematical model to account for differences between the GRAIL and GRACE timing systems. Additionally, we must consider general relativity effects when bounding errors in the LGRS measurement model.

The timing of an LGRS measurement has two components. First, absolute timing knowledge allows us to properly assign a LGRS measurement to a position around the moon. Secondly, the relative timing knowledge between the onboard LGRS clocks allows the proper alignment of the LGRS phase measurements of both spacecraft at a common coordinate time. This alignment is needed to form the DOWR measurement, which is processed by the science team.

For GRACE the absolute and relative timing is determined from the onboard GPS measurements [6]. On the other hand the GRAIL absolute timing is determined by the DSN and the relative timing is determined by the TTS onboard GRAIL. The main difference between the GRAIL and GRACE timing is that for GRACE the absolute and relative timing is available continuously during an orbit, while for GRAIL the relative timing is available continuously but the absolute timing is only available when the GRAIL spacecraft is visible from the Earth and tracked by the DSN. Furthermore, even when the DSN tracks a spacecraft, we only generate an absolute timing measurement once every 10 minutes. Based on planned DSN tracking and lunar orbit geometry, we expect less than 100 absolute timing measurements per day.

Given the limited absolute timing information for the GRAIL mission, the LGRS measurement model is re-derived and formulated in terms of the relative and absolute timing error. For this discussion the formulation and notation in Kim [4] was chosen because the Kim [4] formulation was in terms of the total time tag error. The Brooks [8] formulation used time tag errors after an estimate of the time tag correction was applied. Furthermore both Kim [4] and Brooks [8] assume that the time tag errors of the phase measurements are small in the development of the LGRS measurement model. However, due to limited absolute timing information, the time tag error of the phase measurements may not be small and a bias may exist in the relative timing as well. In the re-derivation of the LGRS measurement model a second order expansion is used for the phase approximation to assess the LGRS measurement errors due to larger phase time tag errors and a bias in the relative timing.

Finally the LGRS measurement model described in Kim [4] and Brooks [8] did not consider general relativity. In this derivation the treatment of the light time is such that an assessment of not modelling general relativity on the LGRS measurement can be made and the effect of general relativity on phase and clock drift is considered as well. Under the assumptions and conditions described the derivation of the LGRS measurement model is as follows.

The single carrier frequency phase measurement for a spacecraft at a given coordinate time t can be written as

$$\varphi_1^2(t+\Delta t_1) = \varphi_1(t+\Delta t_1) - \varphi^2(t+\Delta t_1) + N_1^2 + d_1^2 + \epsilon_1^2 \tag{1}$$

$$\varphi_2^1(t+\Delta t_2) = \varphi_2(t+\Delta t_2) - \varphi^1(t+\Delta t_2) + N_2^1 + d_2^1 + \epsilon_2^1 \tag{2}$$

where

$$\begin{aligned}
\varphi_1^2(t+\Delta t_1) &= \text{differential phase measurement at spacecraft 1} \\
\varphi_2^1(t+\Delta t_2) &= \text{differential phase measurement at spacecraft 2} \\
t &= \text{coordinate time} \\
\Delta t_1, \Delta t_2 &= \text{time tag error at spacecraft 1 and 2 with respect to t} \\
\varphi_1(t+\Delta t_1) &= \text{spacecraft 1 reference phase} \\
\varphi^2(t+\Delta t_1) &= \text{received phase transmitted by spacecraft 2} \\
\varphi_2(t+\Delta t_2) &= \text{spacecraft 2 reference phase} \\
\varphi^1(t+\Delta t_2) &= \text{received phase transmitted by spacecraft 1} \\
N_1^2, N_2^1 &= \text{integer ambiguities} \\
d_1^2, d_2^1 &= \text{phase shift due to instrument, offset, multipath, etc.} \\
\epsilon_1^2, \epsilon_2^1 &= \text{random measurement noise}
\end{aligned}$$

It should be noted that the phase delay due to the ionosphere has been omitted from the formulation described in [4] because of the lack of a lunar ionosphere.

Each phase consists of the reference phase $\overline{\varphi}_i$ which corresponds to the constant reference frequency, and the phase error $\delta\varphi_i(t)$ due to oscillator drift, frequency instability and phase change due to general relativity and hence may be written as:

$$\varphi_1(t) = \overline{\varphi}_1(t) + \delta\varphi_1(t) \tag{3}$$

$$\varphi_2(t) = \overline{\varphi}_2(t) + \delta\varphi_2(t) \tag{4}$$

Additionally, the received phase $\varphi^i(t)$ can be identified with the transmitted phase φ_i:

$$\varphi^i(t) = \varphi_i(t - \tau_j^i) \tag{5}$$

where τ_j^i is the light time from the i-th satellite to the j-th satellite. It should be noted that the light times of the two phase signals are different from each other since the two satellites are moving. Substituting (3), (4), (5) into (1) and (2) yields

$$\begin{aligned}\varphi_1^2(t+\Delta t_1) &= \overline{\varphi}_1(t+\Delta t_1)+\delta\varphi_1(t+\Delta t_1)- \\ &\quad \overline{\varphi}_2(t+\Delta t_1-\tau_1^2)-\delta\varphi_2(t+\Delta t_1-\tau_1^2)+ \\ &\quad N_1^2+d_1^2+\epsilon_1^2 \end{aligned} \tag{6}$$

$$\begin{aligned}\varphi_2^1(t+\Delta t_2) &= \overline{\varphi}_2(t+\Delta t_2)+\delta\varphi_2(t+\Delta t_2)- \\ &\quad \overline{\varphi}_1(t+\Delta t_2-\tau_2^1)-\delta\varphi_1(t+\Delta t_2-\tau_2^1)+ \\ &\quad N_2^1+d_2^1+\epsilon_2^1 \end{aligned} \tag{7}$$

Adding (6) and (7) gives the DOWR phase measurement $\Theta(t)$ definition:

$$\begin{aligned}\Theta(t) &\equiv \varphi_1^2(t+\Delta t_1)+\varphi_2^1(t+\Delta t_2) \\ &= \overline{\varphi}_1(t+\Delta t_1)-\overline{\varphi}_1(t+\Delta t_2-\tau_2^1)+\overline{\varphi}_2(t+\Delta t_2)-\overline{\varphi}_2(t+\Delta t_1-\tau_1^2) \\ &\quad \delta\varphi_1(t+\Delta t_1)-\delta\varphi_1(t+\Delta t_2-\tau_2^1)+\delta\varphi_2(t+\Delta t_2)-\delta\varphi_2(t+\Delta t_1-\tau_1^2)+ \\ &\quad (N_1^2+N_2^1)+(d_1^2+d_2^1)+(\epsilon_1^2+\epsilon_2^1) \end{aligned} \tag{8}$$

Unlike in Kim [4], we choose to approximate our phase measurements by a second order expansion about coordinate time t. Thus the second order expansion for phase terms in equation (8) about the coordinate time t may be written as

$$\overline{\varphi}_i(t+\Delta t_i) = \overline{\varphi}_i(t)+\dot{\overline{\varphi}}_1(t)\Delta t_i+\frac{1}{2}\ddot{\overline{\varphi}}_1(t)\Delta t_i^2 \tag{9}$$

$$\delta\varphi_i(t+\Delta t_i) \approx \delta\varphi_i(t)+\delta\dot{\varphi}_i(t)\Delta t_i+\frac{1}{2}\delta\ddot{\varphi}_1(t)\Delta t_i^2+\Delta_i \tag{10}$$

where $\dot{\overline{\varphi}}_1(t)$ is the constant nominal frequency f_i and $\ddot{\overline{\varphi}}_1(t)=0$ because $\dot{\overline{\varphi}}_1(t)$ is per definition a constant. $\delta\dot{\varphi}$ is the combined frequency offset δf due to the LGRS clock frequency offset and general relativity . The $\delta\ddot{\varphi}_1(t)$ represents the frequency drift $\delta\dot{f}_i$, which consists of a linear fit to the oscillator frequency and the frequency drift due general relativity. The last term Δ_i in equation (8) represents the phase error resulting from integrating the frequency residuals of the linear fit of the oscillator frequency. To assess the size Δ_i we used five minute spaced GRACE clock solutions. From these clock solutions, the clock drift was computed for each five minute sample as well as a one hour clock drift using only hourly samples. The hourly clock drift estimates were then differenced with the half our clock drift estimates from the five minute samples. The size of Δ_i was then computed by multiplying the speed of light, the half hour clock drift difference and the light time of 10^{-3} sec. of Based on the GRACE clocks, the size of Δ_i results in range errors less than 1 micro meter. Assuming the GRACE and GRAIL USOs are similar then Δ_i can be neglected. Thus the different phase and phase error terms in equation (8) can be written as the following second order expansions:

$$
\begin{aligned}
\overline{\varphi}_i(t+\Delta t_i) &= \overline{\varphi}_i(t) + f_i \Delta t_i && (11)\\
\overline{\varphi}_i(t+\Delta t_j - \tau_j^i) &= \overline{\varphi}_i(t) + f_i(\Delta t_j - \tau_j^i) && (12)\\
\delta\varphi_i(t+\Delta t_i) &\approx \delta\varphi_i(t) + \delta f_i \Delta t_i + \frac{1}{2}\delta \dot{f}_i \Delta t_i^2 && (13)\\
\delta\varphi_i(t+\Delta t_j - \tau_j^i) &\approx \delta\varphi_i(t) + \delta f_i(\Delta t_j - \tau_j^i) + \frac{1}{2}\delta \dot{f}_i (\Delta t_j - \tau_j^i)^2 && (14)
\end{aligned}
$$

Substituting (11),(12),(13),(14) into (8) and combining where possible in terms of the relative timing error $(\Delta t_1 - \Delta t_2)$, equation (8) can be written as:

$$
\begin{aligned}
\Theta(t) \approx\ & (f_1\tau_2^1 + f_2\tau_1^2) + (\delta f_1 \tau_2^1 + \delta f_2 \tau_1^2) + \\
& (f_1 - f_2)(\Delta t_1 - \Delta t_2) + (\delta f_1 - \delta f_2)(\Delta t_1 - \Delta t_2) + \\
& (\delta \dot{f}_1 \Delta t_2 - \delta \dot{f}_2 \Delta t_1)(\Delta t_1 - \Delta t_2) + \frac{1}{2}(\delta \dot{f}_1 + \delta \dot{f}_2)(\Delta t_1 - \Delta t_2)^2 + \\
& \frac{1}{2}\delta \dot{f}_1 (2\Delta t_2 - \tau_2^1)\tau_2^1 + \frac{1}{2}\delta \dot{f}_2 (2\Delta t_1 - \tau_1^2)\tau_1^2 + \\
& (N_1^2 + N_2^1) + (d_1^2 + d_2^1) + (\epsilon_1^2 + \epsilon_2^1) \qquad (15)
\end{aligned}
$$

The first term of equation (15) represents the true phase measurement and all the other terms represent errors due to phase time tag errors, general relativity, phase noise, frequency offset of the oscillator and random phase noise. The first four terms are identical to the terms described in Kim [4] who used a linear expansion. All other terms are the result from the second order expansion and many terms are a function of the relative timing error $(\Delta t_1 - \Delta t_2)$. The second order expansion also shows a dependency on the absolute timing error terms Δt_1 and Δt_2. It should be noted the DOWR phase has an unknown bias due to the fact that the phase integer ambiguities $(N_1^2 + N_2^1)$ are not known. Thus the resulting DOWR range measurement has an unknown bias. In section 3.2 the performance of the GRAIL timing system will be discussed and an assessment of the LGRS measurement errors

3.2 LGRS DOWR measurement errors due to expected timing performance of the GRAIL system

This subsection describes the LGRS measurement errors due to expected timing performance of the GRAIL system and the effects of general relativity for the GRAIL system in lunar orbit as described in section 4.1. The expected timing performance listed in Table 2 is used to assess the error terms in the DOWR measurement as described in equation (15). The error assessment is made in terms of the DOWR which can be defined from $\Theta(t)$ in equation (15) as

$$R_{dowr}(t) \equiv c\frac{\Theta(t)}{f_1 + f_2} \qquad (16)$$

Table 2: Expected timing performance of the GRAIL timing system

Timing	Bias (seconds)	Variability (seconds) (1σ)
absolute (Δt_i) i=1,2	1×10^{-1}	1×10^{-4}
relative ($\Delta t_1 - \Delta t_2$)	1×10^{-7}	9×10^{-11}

where c is the speed of light. The error terms in equation (15) will be multiplied by a factor

$$\frac{c}{f_1 + f_2} \approx \frac{3 \times 10^8}{6 \times 10^{11}} = \mathcal{O}(10^{-3}) \tag{17}$$

to assess the error terms in range.

The first term of equation (15) only represents the true phase measurement for the nominal constant frequencies f_1 and f_2. The second term of equation (15) represents the part of the true measurement due to the oscillator frequency offset plus drift and the frequency drift due to general relativity. When the actual measurement is made, the actual frequencies $(f_1 + \delta f_1)$ and $(f_2 + \delta f_2)$ are used. For GRAIL we have a direct measurement of $(f_1 + \delta f_1)$ and $(f_2 + \delta f_2)$ using RSR with an accuracy of 10^{-3} Hz at X-band frequency. Therefore we need to combine the first and second term of equation (15) to derive the total truth phase measurement. We start by defining:

$$f_1 + \delta f_1 \quad = \quad f_1 + \delta \bar{f}_1 + \varepsilon f_1 = \bar{f}_1 + \varepsilon f_1 \tag{18}$$

$$f_2 + \delta f_2 \quad = \quad f_2 + \delta \bar{f}_2 + \varepsilon f_2 = \bar{f}_2 + \varepsilon f_2 \tag{19}$$

where $\bar{f}_1$, $\bar{f}_2$ are the noise free frequency measurements by the RSR and $\delta \bar{f}_1$, $\delta \bar{f}_2$ represent the best estimate of the actual frequency offset plus a linear drift of the oscillator and the frequency offset due to general relativity. The RSR frequency measurement errors are described by εf_1 and εf_2. Substitution of equations (18) and (19) into the first two terms of equation (15) plus converting to DOWR range, results in

$$\frac{c}{\bar{f}_1 + \bar{f}_2}\Theta(t) \quad = \quad \frac{c}{\bar{f}_1 + \bar{f}_2}(\bar{f}_1 \tau_2^1 + \bar{f}_2 \tau_1^2) + \frac{c}{\bar{f}_1 + \bar{f}_2}(\varepsilon f_1 \tau_2^1 + \varepsilon f_2 \tau_1^2) \tag{20}$$

Using the RSR frequency accuracy of 10^{-3} Hz, Ka frequency $= 32 \times 10^{-9}$ Hz and a light time of 10^{-3} sec then the second term of equation (20)

$$\frac{c}{\bar{f}_1 + \bar{f}_2}(\varepsilon f_1 \tau_2^1 + \varepsilon f_2 \tau_1^2) \approx \frac{3 \times 10^8}{32 \times 10^9} \times 2 \times 10^{-3} \times 10^{-3} = \mathcal{O}(10^{-7}) \quad \text{meter} \tag{21}$$

can be neglected. Now we rewrite the light times τ_1^2 and τ_2^1 as:

$$\tau_2^1 \quad = \quad \tau + \delta \tau_2^1 \tag{22}$$

$$\tau_1^2 \quad = \quad \tau + \delta \tau_1^2 \tag{23}$$

where τ is the euclidean instantaneous light time. Substitution of equations (22) and (23) into equation (20) yields

$$\begin{aligned}\frac{c}{\bar{f}_1+\bar{f}_2}\Theta(t) &\approx \frac{c}{\bar{f}_1+\bar{f}_2}(\bar{f}_1+\bar{f}_2)\tau + \frac{c}{\bar{f}_1+\bar{f}_2}(\bar{f}_1\delta\tau_2^1+\bar{f}_2\delta\tau_1^2) \\ &\equiv c\tau+\rho_{TOF} \\ &= \rho+\rho_{TOF}\end{aligned} \tag{24}$$

where $\delta\tau_j^i$ are the differences between the actual light time from satellite i to j (including general relativity) and the euclidean instantaneous light time τ. ρ is the euclidean instantaneous inter-satellite range. The ρ_{TOF} represents the range change due to spacecraft motion and general relativity. For GRACE the ρ_{TOF} was approximated in Kim [4] with sufficient accuracy. However, most assumptions for the GRACE approximation in [4] were invalid for GRAIL. Therefore the MIRAGE software is used for the ρ_{TOF} by computing the one way ranges between the spacecraft which takes into account satellite motion and general relativity without approximations. Before GRAIL becomes operational, we will need to verify that our MIRAGE calculation of τ_j^i has an accuracy of better than 0.01 ps, because if our light time calculation has error less than 0.01 pico seconds, then the DOWR error due to light time errors $\varepsilon\tau_2^1$ and $\varepsilon\tau_1^2$ can be written as:

$$c\frac{(\bar{f}_1\varepsilon\tau_2^1+\bar{f}_2\varepsilon\tau_1^2)}{\bar{f}_1+\bar{f}_2} \approx \mathcal{O}(10^{-6}) \text{ meter} \tag{25}$$

which can neglected.

Terms three and four of equation (15) depends on the relative synchronization error $(\Delta t_1 - \Delta t_2)$. The expected relative synchronization performance is listed in Table 2. It should be noted that the bias in the relative synchronization does not add errors to the DOWR measurement but adds an additional bias to the DOWR measurement. Therefore, we can only evaluate the variability in relative clock synchronization when bounding terms four and five of equation (15). Again we will combine terms three and four of equation (15) to rewrite these terms with the observed frequencies from the RSR measurement. Using definitions (18) and (19) plus converting to DOWR range we may write terms three and four as:

$$c\frac{(\bar{f}_1-\bar{f}_2)}{\bar{f}_1+\bar{f}_2}(\Delta t_1-\Delta t_2) + \frac{c}{\bar{f}_1+\bar{f}_2}(\varepsilon f_1-\varepsilon f_2)(\Delta t_1-\Delta t_2) \tag{26}$$

We know for GRAIL that the baseband frequency $(\bar{f}_1-\bar{f}_2)\approx 6\times10^5$ Hz and the Ka frequency $\bar{f}_i \approx 32\times10^9$ Hz. Thus we can bound the first term of equation (26) by:

$$c\frac{(\bar{f}_1-\bar{f}_2)}{\bar{f}_1+\bar{f}_2}(\Delta t_1-\Delta t_2) \approx 3\times10^8\frac{6\times10^5}{6\times10^{10}}\times 9\times10^{-11} = \mathcal{O}(10^{-6}) \text{ meter} \tag{27}$$

I should be noted that the $(\bar{f}_1-\bar{f}_2)$ is not constant in this context unlike the derivation in Kim [4]. Therefore any variability will have an effect due to the bias in $(\Delta t_1-\Delta t_2)$. However, based on the expected clock performance we do not expect a large effect on the

LGRS measurement. Furthermore, the level-2 processing will have model parameters to observe drifts in the LGRS measurement in case the LGRS clock does not have the expected performance. Finally, for term one in equation (26), the differential effect of general relativity on the frequency of both GRAIL clocks can be neglected because the frequency difference is < 3 Hz based on the maximum relative clock drift in figure 3. The second term in equation (26) may be bounded by

$$\frac{c}{\bar{f}_1+\bar{f}_2}(\varepsilon f_1-\varepsilon f_2)(\Delta t_1-\Delta t_2) \approx 2\times 3\times 10^8\frac{10^{-3}}{6\times 10^{10}}\times 9\times 10^{-11} = \mathcal{O}(10^{-14}) \text{ meter} \quad (28)$$

Before bounding term five of equation (15) we first rewrite this term as

$$\begin{aligned}\frac{c}{f_1+f_2}(\delta\dot{f}_1\Delta t_2-\delta\dot{f}_2\Delta t_1)(\Delta t_1-\Delta t_2) &= \frac{c}{f_1+f_2}(\delta\dot{f}_1-\delta\dot{f}_2)\Delta t_2(\Delta t_1-\Delta t_2)+ \\ &\quad \frac{c}{f_1+f_2}\delta\dot{f}_1(\Delta t_1-\Delta t_2)^2 \qquad (29)\end{aligned}$$

The first term of equation (29) may be bounded

$$\begin{aligned}\frac{c}{f_1+f_2}(\delta\dot{f}_1-\delta\dot{f}_2)\Delta t_2(\Delta t_1-\Delta t_2) &\approx 2\times 3\times 10^8\frac{10^{-9}}{6\times 10^{10}}\Delta t_2\times 10^{-7} \\ &= \Delta t_2\times\mathcal{O}(10^{-17}) \text{ meter} \qquad (30)\end{aligned}$$

For this bound a $\delta\dot{f}_i < 10^{-9}$ was used based on performance of the GRACE clocks. Even though the bound is a function of the time tag error Δt_2, the error it self is $\ll 10^{-6}$ meter for large time tag errors. The second order term of equation (29) and term six in equation (15) can both be bounded by $(\Delta t_1-\Delta t_2)^2$

$$c\frac{(\Delta t_1-\Delta t_2)^2}{f_1+f_2} \approx 3\times 10^8\frac{10^{-14}}{6\times 10^{10}} = \mathcal{O}(10^{-16}) \text{ meter} \quad (31)$$

Since all the multipliers for the $(\Delta t_1-\Delta t_2)^2$ terms in equation (15) are $\ll 1$, we can neglect all these terms.

Finally terms seven and eight of equation (15) do also depend directly on the absolute time synchronization errors $\Delta t_1, \Delta t_2$ and the time of flight. Only the error bound for term seven of equation (15) is discussed because term eight is similar and can be bounded in the same way. Thus the seventh term can be written as:

$$\begin{aligned}\frac{1}{2}c\frac{\delta\dot{f}_1(2\Delta t_2-\tau_2^1)\tau_2^1}{f_1+f_2} &= c\frac{\delta\dot{f}_1}{f_1+f_2}\tau_2^1\Delta t_2-\frac{1}{2}c\frac{\delta\dot{f}_1}{f_1+f_2}(\tau_2^1)^2 \\ &\approx 3\times 10^8\frac{10^{-9}}{6\times 10^{10}}\Delta t_2-\frac{1}{2}\times 3\times 10^8\frac{10^{-9}}{6\times 10^{10}}\times\mathcal{O}(10^{-6}) \\ &= 5\times 10^{-12}\Delta t_2+\mathcal{O}(10^{-17}) \text{ meter} \qquad (32)\end{aligned}$$

which shows that even for an absolute synchronization error of days the error in the DOWR measurement is less than 10^{-6} meters. For this bound a $\delta\dot{f}_1 < 10^{-9}$ was used based on performance of the GRACE clocks. Furthermore, the $\delta \dot{f}_1 < 10^{-12}$ due to general relativity based on data in figure 2. As listed in Table 2, we do not expected absolute synchronization errors larger than 0.1 seconds, hence the seventh term should not provide a significant error source.

In summary with the expected GRAIL timing performance, we do not expect that the errors in the DOWR measurement are not going to exceed 10^{-6} meters.

4 Algorithms for GRAIL timing

This section describes our development of algorithms for time correlation between LGRS clocks and TDB time. In this section, we describe the modeling of clock drift due to general relativity and the modeling of the LGRS clock drift due to imperfections in the LGRS clock. An important part of the GRAIL timing is the determination of the inter-satellite clock time offset using the TTS system. We derive an algorithm to extract the inter-satellite clock time offset from the measurements of the TTS system. A description will be given of all GRAIL measurements used for timing and how they are used in the computation of the time correlation. Finally, we describe our time correlation algorithm, and our production of time tag corrections for science products.

4.1 Modelling LGRS clock rate due to general relativity

This section describes how the LGRS clock rate due to general relativity is modeled for the GRAIL mission. For this discussion it is assumed that the LGRS clock is perfect thus its drift with respect to coordinate time would be zero if it were infinitely far from any mass. According to the theory of general relativity this perfect clock in lunar orbit will have a clock rate which differs from the TDB clock rate. The change in clock rate due to general relativity is described in [5] and can be written as:

$$\frac{d\tau}{dt} = 1 - \frac{U}{c^2} - \frac{1}{2}\frac{v^2}{c^2} + L \tag{33}$$

where τ is proper time, which is the perfect clock reading in lunar orbit, t is coordinate time (TDB), U is the gravitational potential at the GRAIL spacecraft in lunar orbit, c is the speed of light, v is the Solar-System barycentric velocity of the GRAIL spacecraft and L is a constant value. The value of L is chosen such that the clock rate of TDB is equal to the average clock rate of a clock on the Earth surface at sea level with respect to TDB [5]. Thus L may be written in term of the Solar-System barycentric gravitational potential of a clock at Earth's sea level U_E and the barycentric velocity v_E of that clock [5].

$$L = \frac{1}{c^2} < U_E + \frac{1}{2} v_E^2 > \tag{34}$$

The brackets <> denote a long term average. The reason for selecting this L is that our notion of time is established by atomic clocks on the Earth's surface also known as TAI. Choosing another L would cause a linear trend between TDB and TAI. The DSN uses UTC for timing, which is offset by a constant with respect to TAI but this constant changes every time a new leap second is introduced.

For GRAIL equation (33) is integrated to get proper time as a function of coordinate time, using the GRAIL spacecraft ephemerides and the planetary ephemerides DE421[3] for the ten principle solar bodies (point mass). In this integration $L = 1.550520 \times 10^{-8}$ is used [5]. In Figure 2 the GRAIL clock drift rate due to general relativity is shown

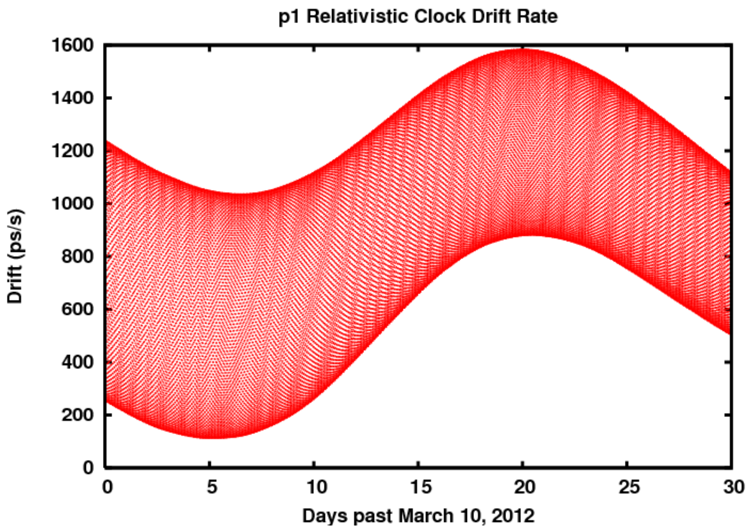

Figure 2: GRAIL clock drift rate due to general relativity during the science phase

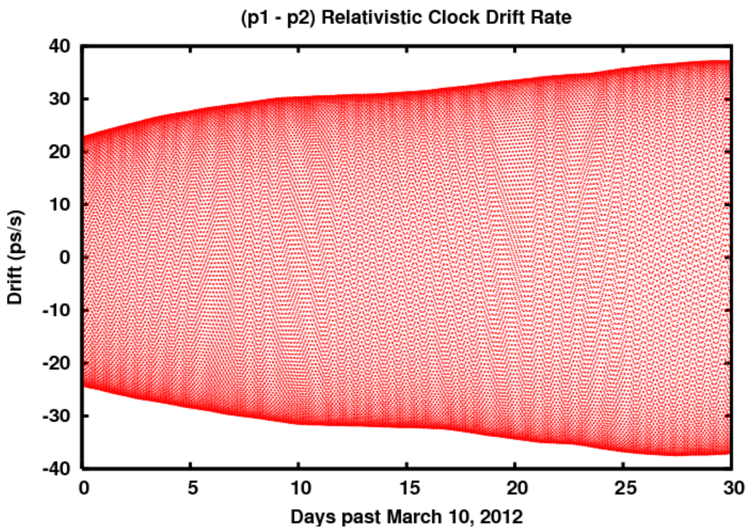

Figure 3: Differential GRAIL clock drift rate due to general relativity during the science phase

for the first 30 days of the GRAIL science phase, where the spacecraft separation starts at 85 km and ends at 225 km. This figure contains the effects of the GRAIL orbit, lunar orbit and the Earth/Moon system orbit around the Sun. The maximum drift rate is about 2×10^{-9} sec/sec. This number will be used in other sections to bound LGRS and TTS measurement errors due to general relativity.

The relative clock drift rate between the two GRAIL clocks for the first 30 days of the GRAIL science phase is shown in Figure 3. From this figure it can be seen that the differential clock drift is less than $\pm 40 \times 10^{-12}$ sec/sec. It should be noted that the differential clock rate is shown for the orbit geometry of the first 30 days of the science phase. However, an orbit geometry to maximize the differential clock drift error puts an upper bound of $\pm 70 \times 10^{-12}$ sec/sec [9]. The upper bound of 70×10^{-12} sec/sec will be used to bound LGRS and TTS measurement errors due to differential clock drift effects of general relativity

4.2 Modelling the LGRS clock offset due to LGRS clock imperfections

This subsection describes the model used for the LGRS clock drift due to imperfections in the LGRS clock. This model does not include LGRS clock drift due to general relativity. The model described in this subsection when added to the general relativity contribution from subsection 4.1 gives the total LGRS clock drift as seen from the TDB time scale.

Unlike GRACE, for which GPS measurements allow us to track clocks continuously, we do not have continuous clock offset measurements for GRAIL. GRAIL may have only 20 LGRS clock offset measurements per day. Moreover, these measurements are biased and a have variability of 20 milli seconds (1 σ). The bias in the clock offset measurement are due to unknown timing delays in the spacecraft. The variability in the clock offset measurement is caused by unknown data packet traffic on the spacecraft bus, which causes a variable transfer time of the time correlation packet from the onboard computer to the spacecraft antenna.

As mentioned in subsections 2.1 and 2.3, we know that the LGRS clock and the clocks that realize the UTC time tags at the DSN are very stable. Therefore, a simple polynomial may be sufficient to describe the LGRS clock drift with respect to TDB, where TDB time tags are determined from UTC using general relativity by integration of equation 33 for a DSN station (see subsection 4.1). It should also be noted that the clock offset measurements also contain a contribution due to general relativity. The general relativity contribution is computed as described in subsection 4.1 and removed from the measurements before fitting a second order polynomial to the measurements.

Using 60 days of clock offset measurements as observed by GRACE, an assessment is made of modeling a second order polynomial for the clock drift error. The GRACE clock serves as a proxy for the LGRS clock since both clocks have similar stability characteristics. The residual for the second order fit is shown in Figure 4.

From Figure 4 it can be seen that throughout the 60 days, the time tag error using a second order polynomial for the LGRS clock drift would be less than 2 micro seconds. Therefore the error in the clock offset is 2 micro seconds or less due to the the non-quadratic LGRS clock drift behavior. The use of higher order polynomials was found not to reduce the time tag errors significantly, therefore we will use a second order polynomial to model the LGRS clock drift due to imperfections in the LGRS clock. If clock drift should prove larger than we hope, we will allow our second order polynomial coefficients to random walk.

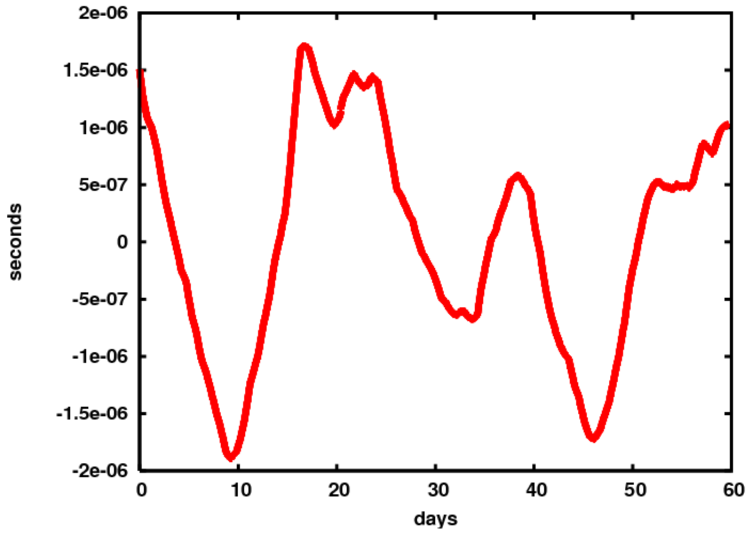

Figure 4: Time tag residuals for GRACE-2 clock after second order polynomial fit

4.3 Algorithm for inter-satellite LGRS clock offset determination

This subsection describes the algorithm and assumptions for the extraction of the inter-satellite LGRS clock offset using the GRAIL TTS inter-satellite ranging system. A complete description of the TTS ranging algorithm can be found in [2]. For this discussion it is assumed that an inter-satellite range measurement is available from both GRAIL spacecraft and that the objective is to determine the LGRS clock offset at a given coordinate time. It should be noted that the relative LGRS clock relationship derived in this section is the inverse of the relative coordinate time clock offset in equation (15) (see section 3.1). The relative LGRS clock offset derived in this section will be used as an observation equation in the determination of the LGRS clock error model described in section 4.4.5. The inversion of the relative LGRS clock offset is part of the complete clock offset model inversion described in section 4.4.6

The objective is to determine the clock offset $O_{12}(T)$ of the two LGRS clocks at a given coordinate time T which may be written as

$$\tilde{t}_{S1}(T) - \tilde{t}_{S2}(T) = O_{12}(T) \tag{35}$$

where $\tilde{t}_1(T)$ is the satellite 1 LGRS clock reading at coordinate time T and $\tilde{t}_2(T)$ is the satellite 2 LGRS clock reading at coordinate time T. Our first assumption is that the interpolation errors of the inter-satellite range are small for both spacecraft. Therefore with interpolation it is possible to change the time of the data and thus change the clock reading without having to restart the range code generation.

At satellite 1 we measure:

$$\tilde{t}^1_{S1}(T_1) - \tilde{t}^1_{S2}(T_1 - \tau^2_1) + \nu \tag{36}$$

where τ^2_1 is the light time of the ranging signal from spacecraft 2 to spacecraft 1 and ν is random noise (assumed to be 10 nsec for GRAIL). In this measurement $\tilde{t}_1(T_1)$ is known and τ^2_1 is unknown. Expanding $\tilde{t}^1_{S2}(T_1 - \tau^2_1)$ about T_1

$$\tilde{t}^1_{S2}(T_1 - \tau^2_1) = \tilde{t}^1_{S2}(T_1) - \dot{\tilde{t}}^1_{S2}(T_1)\tau^2_1 + \Delta^1_{S2} + \mathcal{O}((\tau^2_1)^2) \tag{37}$$

where $\dot{\tilde{t}}^1_{S2}(T_1)$ is a linear fit to the clock drift of spacecraft 1 with respect to coordinate time at T_1. This clock rate is the sum of the clock rate due to general relativity and the LGRS clock error rate. Δ^1_{S2} represents the residuals about the linear fit of the clock drift of spacecraft 1. The last term in equation (37) represents the second order term caused by general relativity. The size Δ^1_{S2} is based on the same GRACE clock analysis used for equation (10) and can be bounded by 10^{-12} sec over the GRAIL light time of $\tau^2_1 = 10^{-3}$ sec. Assuming the GRAIL USO is similar to the GRACE USO, we can neglect Δ^1_{S2}. To bound the last term in equation (37) we determined an upper bound of $\mathcal{O}((10^{-12})$ for $\ddot{\tilde{t}}^1_{S2}$ based on general relativity and shown in figure 2. Thus the last term of equation (37) can be bounded by

$$\mathcal{O}((\tau^2_1)^2) \approx \frac{1}{2}\,\ddot{\tilde{t}}^1_{S2}(T_1)(\tau^2_1)^2 < \mathcal{O}(10^{-12}) \times \mathcal{O}(10^{-6}) = \mathcal{O}(10^{-18}) \text{ sec} \tag{38}$$

Substituting (37) into (36) we measure

$$\tilde{t}^1_{S1}(T_1) - \tilde{t}^1_{S2}(T_1) + \dot{\tilde{t}}^1_{S2}(T_1)\tau^2_1 + \nu = O_{12}(T_1) + \dot{\tilde{t}}^1_{S2}(T_1)\tau^2_1 + \nu \tag{39}$$

Thus with no knowledge of τ^2_1 we know $O_{12}(T_1)$ to 10^{-3} sec since the clock drift rate is

$$\dot{\tilde{t}}^1_{S2}(T_1) = 1 + r \quad \text{where} \quad r = \mathcal{O}(10^{-6}) \ \text{ sec/sec} \tag{40}$$

Parameter r in equation (40) is determined from actual GRACE clock error drift observation and found to be $\mathcal{O}((10^{-6})$ assuming the GRACE and GRAIL USOs are similar. The clock drift due to general relativity is of $\mathcal{O}((10^{-9})$ thus the bound for parameter r is $\mathcal{O}((10^{-6})$. Since the LGRS clocks of satellite 1 and satellite 2 are synchronized to 10^{-3} sec level we can resample R_2. Thus we have now the following range measurements R_1, R_2 (in terms of seconds)

$$R_1 = \tilde{t}^1_{S1}(T_1) - \tilde{t}^1_{S2}(T_1 - \tau^2_1) + \nu_1 \tag{41}$$

$$R_2 = \tilde{t}^2_{S2}(T_2) - \tilde{t}^2_{S1}(T_2 - \tau^1_2) + \nu_2 \tag{42}$$

where $|\tilde{t}^1_{S1}(T_1) - \tilde{t}^2_{S2}(T_2)| < 10^{-3}$ sec and given the clock rate in equation (40) then $T_1 - T_2 = \mathcal{O}(10^{-3})$. Substituting (39) into (41) and (42) results in

$$\begin{aligned} R_1 &= O_{12}(T_1) + \dot{t}^1_{S2}(T_1)\tau^2_1 + \nu_1 && (43) \\ -R_2 &= O_{12}(T_2) - \dot{t}^2_{S1}(T_2)\tau^1_2 - \nu_2 && (44) \end{aligned}$$

Expanding $O_{12}(T_2)$ about T_1 results in

$$O_{12}(T_2) = O_{12}(T_1) + \dot{O}_{12}(T_1)(T_1 - T_2) + \Delta O_{12} \tag{45}$$

where term ΔO_{12} can be neglected using similar arguments as for Δ^1_{S2} in equation (37). Adding equations (43), (44) and substituting (45) results in

$$R_1 - R_2 \approx 2O_{12}(T_1) + \dot{O}_{12}(T_1)(T_1 - T_2) + \dot{t}^1_{S2}(T_1)\tau^2_1 - \dot{t}^2_{S1}(T_2)\tau^1_2 + \nu_1 - \nu_2 \tag{46}$$

To understand the error terms we rewrite the light times τ^2_1 and τ^1_2 as:

$$\begin{aligned} \tau^2_1 &= \tau(T) + \delta\tau^2_1 && (47) \\ \tau^1_2 &= \tau(T) + \delta\tau^1_2 && (48) \end{aligned}$$

where τ is the euclidean instantaneous light time at a given coordinate time T. We may assume that the LGRS clock rates $\dot{t}^2_{S1}(T_2)$, $\dot{t}^1_{S1}(T_1)$ are identical since T_1 and T_2 are already synchronized to 10^{-3} sec after one iteration and using $\dot{t}^1_{S2}$ from equation (38) we can show

$$\dot{t}^2_{S1}(T_2) = \dot{t}^1_{S1}(T_1) + \mathcal{O}(10^{-3}) \times \mathcal{O}(10^{-10}) \text{ sec} \tag{49}$$

Substituting (47),(48),(49) into (46) results in:

$$\begin{aligned} R_1 - R_2 &\approx 2O_{12}(T_1) + \dot{O}_{12}(T_1)(T_1 - T_2) + (\dot{t}^1_{S2}(T_1) - \dot{t}^1_{S1}(T_1))\tau(T_1) + \\ &\quad \dot{t}^1_{S2}(T_1)\delta\tau^2_1 - \dot{t}^1_{S1}(T_1)\delta\tau^1_2 + \nu_1 - \nu_2 && (50) \\ &\approx 2O_{12}(T_1) + \dot{O}_{12}(T_1)(T_1 - T_2) + (\dot{t}^1_{S2}(T_1) - \dot{t}^1_{S1}(T_1))\tau(T_1) + \\ &\quad r_2\delta\tau^2_1 - r_1\delta\tau^1_2 + \delta\tau^2_1 - \delta\tau^1_2 + \nu_1 - \nu_2 && (51) \end{aligned}$$

where

$$\begin{aligned} r_1 &\equiv \dot{t}^1_{S1}(T_1) - 1 && (52) \\ r_2 &\equiv \dot{t}^1_{S2}(T_1) - 1. && (53) \end{aligned}$$

Substituting the expected values for clock drifts (40)), clock synchronization error and light time, then the terms of (51) can be approximated as follows:

$$\begin{aligned} \dot{O}_{12}(T_1)(T_1 - T_2) &= \mathcal{O}(10^{-6}) \times \mathcal{O}(10^{-3}) = \mathcal{O}(10^{-9}) \text{ sec} && (54) \\ (\dot{t}^1_{S2}(T_1) - \dot{t}^1_{S1}(T_1))\tau(T_1) &= \mathcal{O}(10^{-6}) \times \mathcal{O}(10^{-3}) = \mathcal{O}(10^{-9}) \text{ sec} && (55) \\ r_2\delta\tau^2_1 - r_1\delta\tau^1_2 &= \mathcal{O}(10^{-6}) \times \mathcal{O}(10^{-7}) = \mathcal{O}(10^{-13}) \text{ sec} && (56) \\ \delta\tau^2_1 - \delta\tau^1_2 &= \mathcal{O}(10^{-7}) \text{ sec} && (57) \\ \nu_1 - \nu_2 &= \mathcal{O}(10^{-8}) \text{ sec} && (58) \end{aligned}$$

The first observation that can be made is that the clock drift difference multiplied by the light time or synchronization error is determining the size of the of the error terms. It can also be seen that the clock drifts due to general relativity do not play a role because these drifts are nearly identical (difference < 70 psec/sec) and cancel out when computing the terms of equation (51).

The term in equation (54) can be made small by iterating equation (51), so it can be neglected. The iteration procedure will be described later when the algorithm of the inter-satellite clock offset extraction is discussed.

The second term (55) is a variable term because the light time $\tau(T_1)$ varies over the science phase from about 0.3 to 0.7 milli seconds. If the clock drift rates are of order $\mathcal{O}(10^{-6})$ then this term needs to be computed and used as a correction since the size of this term is larger than the expected variability in the TTS measurement of 9×10^{-11} sec (1σ). The relative clock rate due to general relativity is of order $\mathcal{O}(10^{-11})$ and can be neglected

The third and fourth terms (56) represent the change in light time due to satellite motion and general relativity. The size of the third term is well below the expected noise of the TTS measurement of 9×10^{-11} sec (1σ) therefore it can be neglected. The fourth term needs to be computed. Given our expected orbit determination accuracy, residual error should fall at or below $\mathcal{O}(10^{-11})$.

The fifth term (58) is the error due the random noise in the range measurements. Given the size of $\mathcal{O}(10^{-8})$ sec, smoothing is required. We can use carrier phase smoothing since both phase and range measurements are available from the TTS system. The noise on the phase measurements are significant lower than the range measurements. However, the phase measurements have an unknown bias due to the phase ambiguity. With carrier phase smoothing the bias in the phase measurements is computed using the range measurements. The computed phase bias is then used to adjust all phase measurements to form new range measurements with less noise.

The procedure for carrier phase smoothing is as follows. Given are range $R(\tilde{t}_i)$ and phase $\Phi(\tilde{t}_i)$ measurements (expressed in range) at the same observation time $\tilde{t}_i$ which may be written as

$$R(\tilde{t}_i) = \bar{R}(\tilde{t}_i) + \nu_i \qquad i = 1, N \tag{59}$$

$$\Phi(\tilde{t}_i) = \bar{\Phi}(\tilde{t}_i) + K + \varepsilon_i \quad i = 1, N \tag{60}$$

where K is the phase bias, $\bar{R}(\tilde{t}_i)$ is the truth range and $\bar{\Phi}(\tilde{t}_i)$ is the truth phase range. The range and phase noise are denoted by ν_i and ε_i. Equations (59) and (60) can now be used to solve for K under the assumption that $\bar{R}(\tilde{t}_i) = \bar{\Phi}(\tilde{t}_i)$

$$K = \frac{1}{N}\sum_{i=1}^{N}(R(\tilde{t}_i) - \Phi(\tilde{t}_i)) + \frac{1}{N}\sum_{i=1}^{N}(\nu_i - \varepsilon_i) \tag{61}$$

For large N the second term of equation (61) will approach zero. N can be chosen over time intervals for which continuous phase measurements are available. Finally the estimate for K is removed from the phase range measurement to form the new range measurements.

In conclusion the algorithm for extracting the inter-satellite clock offset is as follows. First perform carrier phase smoothing for both spacecraft's range and phase measurements according to

$$\begin{aligned} \tilde{R}_1(\tilde{t}_i^1) &= \Phi(\tilde{t}_i^1) - K_1 \qquad i = 1, N \qquad (62) \\ \tilde{R}_2(\tilde{t}_i^2) &= \Phi(\tilde{t}_i^2) - K_2 \qquad i = 1, N \qquad (63) \end{aligned}$$

where $\tilde{t}_i^1$) and $\tilde{t}_i^2$) are the observation times reported by the LGRS clock. Next iteratively apply equation (51) to minimize the synchronization error in coordinate time $(T_1 - T_2)$ shown in equation(54). The iteration procedure starts by computing the first estimate of the clock offset $\tilde{O}_{12}^1(\tilde{t}_i^1)$

$$\tilde{O}_{12}^1(\tilde{t}_i^1) = \frac{1}{2}(\tilde{R}_1(\tilde{t}_i^1) - \tilde{R}_2(\tilde{t}_i^2)) - \frac{1}{2}(\dot{\tilde{t}}_{S2}^1 - \dot{\tilde{t}}_{S1}^1)\tau \qquad (64)$$

where we assume the LGRS clock drifts $\dot{\tilde{t}}_{S1}^1$, $\dot{\tilde{t}}_{S2}^1$ and the light time τ at observation time $\tilde{t}_i^1$ are known. In general the estimate for the clock offset $\tilde{O}_{12}^1(\tilde{t}_i^1)$ is not equal to $(\tilde{t}_i^1 - \tilde{t}_i^2)$. We can now use the difference to update $\tilde{t}_i^2$ by defining

$$\delta_1 = (\tilde{t}_i^1 - \tilde{t}_i^2) - \tilde{O}_{12}^1(\tilde{t}_i^1) \qquad (65)$$

where 1 denotes the iteration number of the $\tilde{t}_i^2$ update. Using δ_1 to update $\tilde{t}_i^2$) we can now update the calculation for the relative time offset

$$\tilde{O}_{12}^2(\tilde{t}_i^1) = \frac{1}{2}(\tilde{R}_1(\tilde{t}_i^1) - \tilde{R}_2(\tilde{t}_i^2 + \delta_1) - \frac{1}{2}(\dot{\tilde{t}}_{S2}^1 - \dot{\tilde{t}}_{S1}^1)\tau \qquad (66)$$

Thus the complete iteration procedure of M iterations for equation (64) until $\tilde{t}_i^2$ converges, may be written as

$$\begin{aligned} \delta_M &= (\tilde{t}_i^1 - \tilde{t}_i^2) - \tilde{O}_{12}^{M-1}(\tilde{t}_i^1) \qquad (67) \\ \tilde{O}_{12}^{M+1}(\tilde{t}_i^1) &= \frac{1}{2}(\tilde{R}_1(\tilde{t}_i^1) - \tilde{R}_2(\tilde{t}_i^2 + \sum_{k=1}^{M} \delta_k) - \frac{1}{2}(\dot{\tilde{t}}_{S2}^1 - \dot{\tilde{t}}_{S1}^1)\tau \qquad (68) \end{aligned}$$

It should be noted that the convergence of $\tilde{t}_i^2$ depends on the phase range noise ε_i. The evaluation of $\tilde{R}_2(\tilde{t}_i^2 + \sum_{i=1}^{M} \delta_k)$ is done with lagrangian interpolation of the $\tilde{R}_2(\tilde{t}_i^2)$ data.

Depending on the size of the spacecraft's phase range noises ε_i^j, additional smoothing may be needed of the time series after M iterations.

$$\tilde{O}_{12}^M(\tilde{t}_i^1) + \varepsilon_i^1 + \varepsilon_i^2 \qquad (69)$$

In case additional smoothing is needed, a simple box car filter will suffice to attenuate the remaining ε_i^j noise. The derived time series in equation (69) is known as a function of the LGRS observation time $\tilde{t}_i^1$. In the final step the $\tilde{t}_i^1$ time tags are changed to TDB time using the time correlation function described in section 4.4.6.

4.4 Measurements and Algorithm for determination of LGRS clock error model

This subsection describes the measurements and algorithm for the determination of the LGRS clock error model. The LGRS clock error models the observed LGRS clock drift after the clock drift due general relativity has been removed. As described in section 4.2 the LGRS clock error is modelled as a second order polynomial with TDB time as the independent variable. The second order polynomial will model a clock bias, clock drift due to a frequency offset and linear drift in the frequency. The measurements available to estimate the LGRS clock drift model parameters are shown in Table 3.

Table 3: Measurements available for LGRS clock error model determination

Source	Observable	Frequency (seconds)	Measurement Errors (μ, σ)	Availability
DSN	LGRS/DSN time offset	600.0	(0.01, 0.03) sec	DSN uplink (in view)
RSR	SRB X-band frequency	300.0	(1e-5, 1e-3) Hz	RSR tracking (in view)
TTS	LGRS inter-satellite time offset	10.0	(1e-7, 9e-11) sec	continuous

4.4.1 LGRS/DSN time offset measurements

The only direct measurement of the total LGRS clock drift is provided by the DSN, which records the reception time at a DSN station of a time correlation packet sent by the spacecraft. The time correlation packet is only sent by the spacecraft when the DSN has an uplink to the spacecraft, which is planned for 8 hours per day. Another limitation is that the time correlation packet can only be received when the spacecraft is in view of the DSN station. At the beginning and end of the science phase the spacecraft will be occulted by the Moon for tens of minutes every orbit. Therefore, at the beginning and ending of the science phase we may only have 20 to 30 observations/day/spacecraft.

The time correlation provided by the DSN is not a direct time correlation between the LGRS clock and the DSN clock UTC. The DSN time correlation is instead between the OSC and UTC. The time correlation between the LGRS clock and the OSC can be reconstructed on the ground based on the OSC time stamp for the 1 PPS pulse of the LGRS with an accuracy is of 40 $\pm$15 micro seconds. Thus, the total time correlation between the LGRS and UTC is achieved by combining the time correlations from the LGRS/OSC clocks and the time correlation between the OSC/LGRS clocks.

It should also be noted that our DSN/LGRS time offset measurements are not only sparse, but also suffer from a much higher expected bias and sigma than do our GRACE GPS measurements. However, the LGRS/OSC time correlation is known to accuracy of 40 $\pm$15 micro seconds, the time OSC/DSN correlation is not, due variable transmission times of the time correlation packet from the onboard computer to the spacecraft antenna. There may be also unknown timing delays in the transmission of the time correlation packets. For this memo we assume bias delays of up to 100 milli seconds and 30 milli seconds (1σ) for the variability.

4.4.2 RSR frequency measurements

At each DSN complex multiple RSR receivers are deployed to facilitate open loop recording of spacecraft signals for science purposes. For GRAIL the plan is to observe the GRAIL X-band Radio Science Beacon (RSB) which is directly connected to the GRAIL USO. The GRAIL LGRS clock is driven by the USO. Therefore any change in the USO frequency would be directly measured by the RSR and thus a direct measurement of the slope of the LGRS clock drift due to imperfections in the LGRS clock and general relativity.

The tracking plan for the RSR is to observe both GRAIL spacecraft 24 hours a day during the science phase. The RSR can only observe when the GRAIL spacecraft are in view from a given DSN station. In the beginning and end of the science phase, 40 to 50 % of each orbit can not be observed due to occultation by the Moon.

The RSR measurements are expected to have a small frequency bias ($< 10^{-5}$ Hz) and a variability of 10^{-3} Hz (1σ).

4.4.3 TTS inter-satellite time offset measurements

The TTS inter-satellite time offset measurement is the combination of the two GRAIL inter-satellite range measurements as described in subsection 4.3. The main objective for the TTS measurements is to minimize errors in the LGRS DOWR measurements (see section 3.1), however the measurements also provide information of the relative clock drifts between the two LGRS clocks.

The TTS inter-satellite time offset measurements are continuously available and the expected bias is $< 10^{-7}$ seconds and variability of 9×10^{-11} seconds (1σ).

4.4.4 TTS-DTE LGRS/DSN time offset measurements

Twice every two weeks, DSS-24 eavesdrops on TTS inter-satellite signals. These TTS-DTE measurements provide a more accurate measurement of absolute time than time correlation packets, since we don't need to estimate a transmission delay bias.

4.4.5 Algorithm for determination of LGRS clock error model

This subsection describes the algorithm for determination of the LGRS clock error model parameters. This algorithm combines DSN time correlation measurements, RSR X-band RSB frequency measurements, TTS inter-satellite time offset measurements, and TTS-DTE time offset measurements into one estimate for the LGRS clock error.

The algorithm consists of two steps. First, for each measurement the effects of general relativity are computed according to the description presented in subsection 4.1.

The effects of general relativity are then removed from the measurements so that the measurements only contain the effect of frequency bias and drift in the LGRS clocks. For the remainder of this subsection it is assumed that measurements are corrected for effects from general relativity. Second, the second order polynomial coefficients of both LGRS clock error models are simultaneously estimated in a non-causal Kalman filter, with random walks on offset, bias, and drift.

For Grail spacecraft $i = 1, 2$, our Kalman filter moves from time t_k to $t_{k+1} = \delta_k + t_k$ by applying the update equations:

$$\Delta T^i\left(t_{k+1}\right) = \Delta T^i\left(t_k\right) + b_i\left(t_k\right)\delta_k + a_i\left(t_k\right){\delta_k}^2/2 + w_i \tag{70}$$

$$b_i\left(t_{k+1}\right) = b_i\left(t_k\right) + a_i\left(t_k\right)\delta_k + v_i \tag{71}$$

$$a_i\left(t_{k+1}\right) = a_i\left(t_k\right) + u_i \tag{72}$$

where $\Delta T^i\left(t_k\right)$ is the clock offset on spacecraft i at time t_k, b_i and a_i are clock bias and drift, and white noise parameters u_i, v_i, and w_i allow random walk on clock offset, rate, and drift.

DSN time correlation and TTS-DTE observations, adjusted for relativistic effects and time scales, directly measure ΔT^i. RSR frequency observations f^i_{RSR}, scaled by the reference X-band RSB frequency f^i_0, measure clock rate.

$$\frac{f^i_{RSR}}{f^i_0} - 1 = b_i \tag{73}$$

TTS inter-satellite time offset observations measure relative clock offset.

$$\Delta T_{TTS} = \Delta T^1 - \Delta T^2 \tag{74}$$

Simulation results, solving for LGRS clocks as quadratics with offset, bias, and drift random walks parameters constrained to 0, for a 90 day science period, suggest that the quadratic and linear terms of the LGRS clock drift models are well determined assuming the measurement errors in Table 3 and appropriate data type weighting to account for the number of measurements available per data type. In our simulation the truth clock drift model also included the non-quadratic clock drift behavior as shown in Figure 4 extended to 90 days. We did not include TTS-DTE measurements in our simulation, hence the constant terms in our quadratics were only known to about 0.5 ms, due to the fact that the DSN time correlation measurements are sparse and have much higher noise than the RSR and TTS measurements. It should also be noted that the assumed unknown 100 milli second spacecraft timing bias is not included in this analysis because there is no observability of this bias with measurements available. Thus a more accurate timing bias for both LGRS clocks is at the level of 100 milli seconds. Including TTS-DTE measurements should reduce these constant biases by several orders of magnitude. In either case, this offset can be estimated with the lunar gravity field estimation using the LGRS measurements.

4.4.6 Computation of LGRS/TDB time correlation

This section describes the computation of LGRS to TDB time correlation. The LGRS/TDB time correlation has two objectives. First to provide an absolute TDB time tag for the LGRS measurements. Second, to preserve the relative LGRS inter-satellite relative clock TTS information which minimizes the LGRS DOWR errors due to timing errors.

The LGRS clock error models estimated in section 4.4.5 provide a time correlation between an LGRS time $\tilde{t}$ and coordinate time t, ignoring general relativity corrections. The total time correlation may be formed by combining general relativity, the clock error model from section 4.4.5 and adding a correction term $\delta TTS(t)$, which insures that the difference between the two clock correlation models is the TTS measurement. Thus the total time correlations may be written as:

$$\delta TTS(t) = \Delta T_{tts}(t) - \Delta T^1_{gr}(t) + \Delta T^2_{gr}(t) - \Delta T^1(t) + \Delta T^2(t) \tag{75}$$

$$\tilde{t} = t + \Delta T^1_{gr}(t) + \Delta T^1(t) + \frac{1}{2}\delta TTS(t) = t + C_1(t) \tag{76}$$

$$\tilde{t} = t + \Delta T^2_{gr}(t) + \Delta T^2(t) - \frac{1}{2}\delta TTS(t) = t + C_2(t) \tag{77}$$

where $\Delta T_{tts}(t)$ is the measured clock offset at coordinate time t, $\Delta T^1_{gr}(t)$, $\Delta T^2_{gr}(t)$ are the clock offsets due general relativity at coordinate time t and $C_1(t)$, $C_2(t)$ are the time correlation functions from coordinate time t to LGRS time $\tilde{t}$. Note that time correlations $C_1(t)$, $C_2(t)$ preserve the measured relative clock offset, which is the second objective.

For observations we actually need the inverse time correlation functions $C_1^{-1}(\tilde{t})$, $C_2^{-1}(\tilde{t})$ because we want to find the time correlation from LGRS time to TDB. The inverse time correlation functions $C_1^{-1}(\tilde{t})$, $C_2^{-1}(\tilde{t})$ are determined numerically because the time correlation functions $C_1(t)$, $C_2(t)$ are not linear. The time correlation equations for operations may then be written as

$$t = \tilde{t} + C_1^{-1}(\tilde{t}) \tag{78}$$

$$t = \tilde{t} + C_2^{-1}(\tilde{t}) \tag{79}$$

The time series of the inverse time correlation functions $C_1^{-1}(\tilde{t})$, $C_2^{-1}(\tilde{t})$ are stored in the the GRAIL Level-1 product CLK1B. Note that the time correlation functions $C_1(t)$, $C_2(t)$ are nearly linear and therefore any error bound analysis for the components of $C_1(t)$, $C_2(t)$ are applicable for the inversion time correlation functions $C_1^{-1}(\tilde{t})$, $C_2^{-1}(\tilde{t})$.

5 Summary

In this memorandum we have shown, with the current design of the GRAIL timing system and the expected performance of the GRAIL timing system, that the LGRS measurement error is less than the 1 micro meter. Timing Effects due to general relativity were included and where necessary, corrections were added in the algorithms to determine the instantaneous range from the DOWR measurement . The processing of the TTS to determine the relative timing offset of the LGRS clocks is crucial to achieve the 1 micro meter error in the LGRS measurement. It was determined that the relative timing algorithm only requires the two range measurements from the TTS and a correction for the LGRS clock drifts to achieve the relative timing performance after carrier smoothing is performed.Furthermore, we have demonstrated that absolute timing knowledge plays only a minute role in the LGRS measurement errors due to timing errors. Finally we have demonstrated that the algorithm to combine 90 days of the DSN time correlation, TTS and RSR measurements to compute the LGRS clock to TDB time correlation meets the science relative timing requirement and absolute timing knowledge < 10 milli seconds.

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7 Acronyms

DOWR Dual One Way Range 1

DSN Deep Space Network 1

GRACE Gravity Recovery and Climate Experiment 1

GRAIL Gravity Recovery and Interior Laboratory 1

GPS Global Positioning System 1

LGRS Lunar Gravity Ranging System 1

OSC Onboard Spacecraft Clock 4

PPS Pulse Per Second 5

RSB Radio Science Beacon 22

RSR Radio Science Receiver 1

TAI International Atomic Time 6

TDB Barycentric Dynamical Time 1

TTS Time Transfer System 1

USO Ultra Stable Oscillator 3

UTC Coordinated Universal Time 2

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