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Description of Method

The temperature and density of the dayside calcium exosphere are estimated by fitting the apparent column densities from individual limb scans with a model based on the framework of Chamberlain (1963) with a correction that accounts for the finite photoionization lifetime of calcium atoms based on Cui et al. (2008). Calcium limb scan data within ∼ 3000 km of the surface are fit with this model. The data product gives the means and standard deviations of these fits as a function of true anomaly (TAA) in 5° increments.

The apparent column density N (integrated column along the line of sight) are approximated from the radiance $4\pi I$ measured by MASCS UVVS using the formula

$$N_{ap} = 10^9 \frac{4\pi I}{g} \tag{1}$$

where N_{ap} is the apparent illuminated column density measured in cm^{-2}, $4\pi I$ is measured in kR, and g is the "g-value," the product of the photon flux at the doppler-shifted emission wavelength and the absorption probability per atom calculated by Killen et al. (2009). Since we have only considered dayside limb scans, the lines of sight are entirely in sunlight. The g-value is a function of the radial velocity of the atom relative to the Sun; here we have assumed that the atoms are stationary relative to Mercury and can therefore use Mercury's radial velocity relative to the Sun to calculate g. Burger et al. (2014) have shown that this assumption introduces a systematic error into the calculation of g due to the fact that atoms have a range of speeds relative to Mercury. The influence of this error is greatest near Mercury perihelion and aphelion, and produces a greater apparent column density at these true anomalies that was actually observed by UVVS. The surface densities (and possibly temperatures) derived near perihelion and aphelion should therefore be used with caution.

The density as a function of distance from Mercury's center is calculated as given by Cui et al. (2008):

$$n(r) = n_0 e^{-(\lambda_0 - \lambda)} \left[\zeta'_{bal}(\lambda) + \zeta'_{esc}(\lambda)\right] \tag{2}$$

$$\lambda = \frac{GMm}{kTr} \tag{3}$$

where G is the gravitational constant, M is Mercury's mass, m is the mass of a Ca atom, k is Boltzmann's constant, T is the source temperature (or the exospheric temperature at Mercury's surface), r is the distance from Mercury's center, and the subscript 0 refers to values at the surface ($r_0 = R_{\text{Merc}} = 2440$ km). ζ'_{bal} and ζ'_{esc} are the partition functions for ballistic and escaping atoms, respectively, modified to include photoionization of atoms in the exosphere. Satellite atoms (those which neither return to the surface ballistically nor escape) are not included for a surface-bounded exosphere such as at Mercury. The partition functions are given by:

$$\begin{aligned}\zeta'_{bal}(\lambda) = \frac{1}{\pi^{1/2}} \Bigg[& \int_{-\lambda^{1/2}}^{-\xi_1} d\xi \int_0^{\nu_2} d\nu e^{-\xi^2-\nu-\frac{t(\lambda,\xi)}{\tau_{loss}}} + \int_{-\xi_1}^{\xi_1} d\xi \int_0^{\nu_1} d\nu e^{-\xi^2-\nu-\frac{t(\lambda,\xi)}{\tau_{loss}}} \\ & + \int_{\xi_1}^{\lambda^{1/2}} d\xi \int_0^{\nu_2} d\nu e^{-\xi^2-\nu-\frac{t(\lambda,\xi)}{\tau_{loss}}} \Bigg]\end{aligned} \tag{4}$$

$$\zeta'_{esc}(\lambda) = \frac{1}{\pi^{1/2}} \left[\int_{\xi_1}^{\lambda^{1/2}} d\xi \int_{\nu_2}^{\nu_1} d\nu e^{-\xi^2-\nu-\frac{t(\lambda,\xi)}{\tau_{loss}}} + \int_{\lambda^{1/2}}^{\infty} d\xi \int_0^{\nu_1} d\nu e^{-\xi^2-\nu-\frac{t(\lambda,\xi)}{\tau_{loss}}} \right] \tag{5}$$

The integrals are performed numerically with the limits:

$$\xi_1^2 = \lambda(1 - \lambda/\lambda_0) \tag{6}$$

$$\nu_1 = \frac{\lambda^2(\lambda_0 - \lambda - \xi^2)}{\lambda_0^2 - \lambda^2} \tag{7}$$

$$\nu_2 = \lambda - \xi^2 \tag{8}$$

The calcium photoionization rate at 1 AU is given by Huebner and Mukherhee (2011). At Mercury's orbit, the photoionization lifetime in seconds, τ_{loss}, is given by:

$$\tau_{loss} = \frac{R(\mathrm{TAA})^2}{7 \times 10^{-5}\,\mathrm{s}^{-1}} \tag{9}$$

where $R(\mathrm{TAA})$ is Mercury's distance from the Sun as a function of true anomaly. $t(\lambda, \xi)$ is given by:

$$t_+(\lambda, \xi) = \frac{GMm}{kTv_{th}} \int_{\lambda}^{\lambda_0} \frac{d\lambda}{\lambda^2(\xi^2 + \lambda)^{1/2}} \tag{10}$$

$$t(\lambda, \xi) = \begin{cases} t & \xi \geq 0 \\ \frac{2GMm}{kTv_{th}} \int_{\lambda_0}^{\lambda} \frac{d\lambda}{\lambda^2(\xi^2+\lambda)^{1/2}} - t_+ & \xi < 0 \end{cases} \tag{11}$$

$$v_{th} = \left(\frac{2kT}{m}\right)^{1/2} \tag{12}$$

The modeled column density is determined by integrating the density from Equation 2 along lines of sight:

$$N_{mod}(T, n_0; r) = \int_{-\infty}^{\infty} n(T, n_0; r')dy \tag{13}$$

where $r' = (r^2 + y^2)^{1/2}$.

The best fit parameters T and n_0, and their uncertainties σ_T and σ_{n_0}, for each limb scan are determined by minimizing the function

$$\chi^2 = \sum \frac{(N_{ap} - N_{mod}(T, n_0))^2}{\sigma_{ap}^2} \tag{14}$$

where σ_{ap} is the uncertainty in the measured values N_{ap}, using Brent's method (Press et al., 2007).

Mercury's calcium exosphere shows a persistent annual variation in its density (Burger et al., 2014). We therefore report the average surface density and temperature as a function of Mercury true anomaly. The limb scans are grouped into bins of 5° true anomaly and 1 hr local time (e.g., a bin contains all the limb scans with true anomaly between i and $i+5°$ and local time between j and $j+1$ hr. The reported values **TEMPERATURE** and **SURFACE_DENSITY** for each true anomaly and local time bin are determined from the means of the best fit parameters of the limb scans in that bin weighted by their uncertainties. The reported values **TEMPERATURE_STDDEV** and **SURFACE_DENSITY_STDDEV** are the standard deviations in σ_T and σ_{n_0}, respectively. These therefore represent the annual variations in these parameters rather than their uncertainties.

There are several caveats that one must be aware of before making use of these results:

1. The Chamberlain models assume the exosphere is produced isotropically at the given temperature. Burger et al. (2012, 2014) have shown that the Ca source is confined to the dawn equatorial region. Therefore, the isotropic assumption is not valid.
2. These models assume that all the calcium is at rest relative to Mercury when estimating the apparent column density from the observed radiance. As shown in Burger et al. (2014), this assumption leads to an exaggerated peak in column density at perihelion (true anomaly = 0°) and a spurious peak at aphelion (true anomaly = 180°). The latter peak is not real and should not be regarded as evidence of a localized peak in the calcium source.

Fields in the Data Product

The absence of a value in any of the following is indicated by the value -1.0. All values are double precision.

TRUE_ANOMALY Mercury's orbital position in degrees. Fit parameters are provided as averages over 5° increments with the middle of that increment listed in the file (e.g., 2.5° covers the true anomaly range 0°-5°).

LOCAL_TIME The local time of the limb scan. Only limb scans within 0.5 hr of the given local time are included in the fit.

SURFACE_DENSITY Average surface density from the modified Chamberlain model fits for all limb scans in the given true anomaly and local time range (units = cm^{-3}).

SURFACE_DENSITY_STDDEV Standard deviation of the surface density fits for all limb scans in the true anomaly and local time range (units = cm^{-3}).

TEMPERATURE Average temperature from the modified Chamberlain model fits for all limb scans the given true anomaly and local time range (units = K).

TEMPERATURE_STDDEV Standard deviation of the temperature fits for all limb scans in the true anomaly and local time range (units = K).

SCALE_HEIGHT Not used for this data product but included for consistency.

START_TIME A string with the UTC time of the first observation used in this data product (format = 'YYYY-MM-DDTHH:MM:SS').

STOP_TIME A string with the UTC time of the last observation used in this data product (format = 'YYYY-MM-DDTHH:MM:SS').

References

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