JET PROPULSION LABORATORY

ENGINEERING MEMORANDUM

314-558

26 February 1993

TO: Distribution

FROM: T. D. Moyer

SUBJECT: Frame Tie Rotation and Nutation Corrections for the ODP

REFERENCES:

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4. Moyer, T. D., "Proposed Changes to ODP Transformation Between Body-Fixed and Space-Fixed Coordinates for the Planets and the Sun," EM 314-271, 16 June 1982.
5. Moyer, T. D., "Time Derivatives and Partial Derivatives of the Body-Fixed to Space-Fixed Coordinate Transformation for the Sun, Planets, and Planetary Satellites," EM 314-379, 22 August 1985.
6. Moyer, T. D., "Change to Parameterization of Body-Fixed to Space-Fixed Coordinate Transformation Matrix Used for all Bodies Except the Earth," IOM 314.5-1005, 23 April 1986.
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I. INTRODUCTION

This memo specifies the formulation changes which are necessary to add the frame tie rotation matrix and the nutation corrections $\delta\psi$ and $\delta\epsilon$ to the ODP. The frame tie rotation matrix relates the radio frame (RF), to which earth centered space-fixed position vectors of tracking stations and unit vectors to quasars are referred, and the planetary ephemeris frame (PEF). The nutation corrections will be added to the nutations in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$ obtained from the 1980 IAU Theory of Nutation.

The nutation corrections will be obtained by interpolating the Earth Orientation Parameter (EOP) file generated by Section 335. This file will contain values of the X and Y coordinates of the earth's true pole, the time differences TAI-UT1 and TAI-UTC, and the nutation corrections $\delta\psi$ and $\delta\epsilon$ at the time argument t (which is UTC). There is no restriction on the spacing of this data but it is expected to be about a day. These EOP parameters, earth fixed station coordinates (in the IERS Terrestrial Reference Frame), quasar coordinates (except three of them which are held fixed and define the IERS Celestial Reference Frame, i.e., the radio frame), and the frame tie rotation matrix are determined by fitting to VLBI and Lunar Lasar Ranging (LLR) data.

The frame tie rotation matrix contains small solve-for rotation angles about three mutually perpendicular axes. The frame tie rotation and the nutation corrections affect mainly links REGRES and PV of the ODP. In order to incorporate these corrections into the ODP formulation, the space-fixed J2000 reference frame must be specified to be the planetary ephemeris frame, the radio frame, or some other specific frame. The choice for the space fixed reference frame is the planetary ephemeris frame, which is different for each planetary ephemeris. The reasons for this selection are given in Section II. The main consequences of using the planetary ephemeris frame for the J2000 space-fixed reference frame are:

1. Position, velocity, and acceleration vectors interpolated from the planetary ephemeris and from satellite ephemerides (which we will consider to be in the planetary ephemeris frame) can be used directly.

2. The spacecraft ephemeris will be numerically integrated in the planetary ephemeris frame. The frame tie rotation will affect the orientation of the earth in this frame and hence the acceleration of a near earth spacecraft due to the earth's harmonic coefficients. However, as explained in Section II, we don't need the partial derivatives of this acceleration with respect to the three frame tie rotation angles.

3. Using the new EOP file places the geocentric space-fixed position, velocity, and acceleration vectors of the tracking station in the radio frame. The frame tie rotation matrix will be used to rotate these vectors into the planetary ephemeris frame. We will calculate the partial derivatives of the position vector in the PEF with respect to the three frame tie rotation angles.

4. The frame tie rotation matrix will also be used to rotate unit vectors to quasars from the radio frame to the planetary ephemeris frame.

Section III lists the models which will be changed due to adding the frame tie rotation and the nutation corrections. It also lists models which could eventually be changed but will not be changed at the present time. Section IV lists the specific calculations in link REGRES which are affected by the model changes listed in Section III. Section V gives the same information for link PV. Changes for other links of the ODP are listed in Section VI.

Adding the frame tie rotation matrix allows us to represent this error source explicitly. Calculation of the partial derivatives of this rotation matrix with respect to its three rotation angles allows us to represent the uncertainty in the frame tie rotation. Since this error source and its associated uncertainty are represented explicitly, they don't have to be absorbed into the earth fixed station location set and its covariance matrix. Note that the absorbed quantities were poor representations of the actual error source and its uncertainty. After implementing the changes specified in this memo, the earth-fixed station locations should be independent of the planetary ephemeris. However, the frame tie rotation angles will be a function of the planetary ephemeris.

A memo describing the EOP file will be written by Bill Folkner and Alan Steppe.

II. SELECTION OF THE PLANETARY EPHEMERIS FRAME FOR THE SPACE FIXED J2000 REFERENCE FRAME

If we were to use the radio frame as the space-fixed reference frame, every position, velocity, and acceleration vector interpolated from the planetary ephemeris, the satellite ephemeris, and the probe ephemeris would have to be rotated from the planetary ephemeris frame to the radio frame. The partial derivatives of all of the position vectors with respect to the solve-for parameters would also have to be rotated to the radio frame. We would also have to calculate the partial derivatives of all of the rotated position vectors with respect to the frame tie rotation angles. It is far easier to rotate the space-fixed geocentric position, velocity, and acceleration vectors of the tracking station from the radio frame to the planetary ephemeris frame. The partial derivatives of the geocentric space-fixed position vector of the tracking station with respect to the solve-for parameters must also be rotated. Also, we need to calculate the partial derivatives of the rotated position vector with respect to the frame tie rotation angles. Fewer additional calculations are required to add the frame tie rotation to the ODP when the planetary ephemeris frame is selected to be the space-fixed J2000 reference frame.

An alternative to the above comparison would be to integrate the spacecraft ephemeris in the radio frame when that frame is selected as the space-fixed frame of reference. However, the perturbing bodies are tied to the planetary ephemeris frame and we would have to calculate dynamic partial derivatives with respect to the frame tie rotation angles. This variation of using the radio frame as the space-fixed reference frame is even less desirable than the one used in the above comparison.

When the spacecraft ephemeris is integrated in the planetary ephemeris frame, the only effect of the frame tie rotation and the nutation corrections is in the earth-fixed to space-fixed transformation matrix. This transformation is used to calculate the acceleration of the spacecraft due to the earth's harmonic coefficients, mascons, drag, albedo, infrared radiation, and solid earth tides. The partial derivatives of these accelerations with respect to the frame tie rotation angles are very complicated, but are only significant (relative to the partials of the space-fixed position vector of the tracking station with respect to the frame tie rotation angles) when the spacecraft is very near the earth. However, a near earth spacecraft orbit is not tied to the planetary ephemeris frame and is free to rotate with the earth as the frame tie rotation angles are varied. Hence, there is no sensitivity to the frame tie rotation angles for a spacecraft near the earth and consequently we don't need to calculate dynamic partials with respect to the frame tie rotation angles in link PV. The ODP user is cautioned not to estimate or consider frame tie rotation angles for a near earth spacecraft. For a deep space probe, where the trajectory is tied to the planetary ephemeris frame, the partial derivatives with respect to the frame tie rotation angles correctly account for the variation in the space-fixed position of the tracking station. Therefore, for a distant probe, the user is free to estimate or consider the frame tie rotation angles.

III. MODELS WHICH WILL BE CHANGED AND MODELS WHICH WILL REMAIN UNCHANGED

A. Introduction

Section B lists the models which will be changed due to adding the frame tie rotation matrix and the nutation corrections. Section C lists models which could be changed but will remain unchanged at the current time. Sections IV, V, and VI list the specific calculations in links REGRES, PV, and other ODP links which will be affected by the model changes given in Section B.

B. Models Which Will be Changed

1. Corrections to Nutation Angles

The nutation in longitude $\Delta\psi$ and the nutation in obliquity $\Delta\epsilon$ are currently evaluated using the 1980 IAU Theory of Nutation. These nutation angles and their time derivatives are obtained by the Rotations Library which interpolates them from the planetary ephemeris file or evaluates the series for these angles input on the GIN file. The time argument is coordinate time ET (in the solar system barycentric frame or geocentric frame). The rotations library must be modified so that whenever the nutation angles are requested, the corrections $\delta\psi$ and $\delta\epsilon$ to $\Delta\psi$ and $\Delta\epsilon$, respectively, are interpolated from the Section 335 Earth Orientation Parameter (EOP) file using UTC as the time argument. This value of the argument can be obtained by converting the ET value of the argument. In the geocentric frame of reference, ET-TAI=32.184s. In the solar system barycentric frame, it is this constant plus the 1.6 ms annual term. The value of TAI-UTC is given on the EOP file. The EOP file will be interpolated using X X Newhall's interpolator. The time derivatives of the six quantities on the file can and should be requested. This will give us

$$\dot{X}, \dot{Y}, (TAI - UT1)^{\cdot}, (TAI - UTC)^{\cdot} = 0, (\delta\psi)^{\cdot} \quad \text{and} \quad (\delta\epsilon)^{\cdot}$$

The derivative $(TAI - UT1)^{\cdot}$ is used in REGRES to calculate $\dot{\theta}$, the derivative of sidereal time with respect to coordinate time ET. The derivatives $\dot{X}$ and $\dot{Y}$ can be used as described in Section 3. The nutation corrections $\delta\psi$ and $\delta\epsilon$ should be added to the nutation angles $\Delta\psi$ and $\Delta\epsilon$ obtained from the 1980 IAU Theory of Nutation. Then, the corrected nutation angles

$$\Delta\psi + \delta\psi \quad \text{and}$$

$$\Delta\epsilon + \delta\epsilon$$

will be used instead of $\Delta\psi$ and $\Delta\epsilon$, respectively. Also, it is necessary to add $(\delta\psi)^{\cdot}$ and $(\delta\epsilon)^{\cdot}$ to $(\Delta\psi)^{\cdot}$ and $(\Delta\epsilon)^{\cdot}$, respectively.

2. **Frame Tie Rotation Matrix**

From Ref. 1, p. 12, the transformation from the planetary ephemeris frame to the radio frame is given by

$$\mathbf{r}_{RF} = R_x R_y R_z \mathbf{r}_{PEF} \tag{1}$$

where $\mathbf{r}_{PEF}$ and $\mathbf{r}_{RF}$ are position vectors with rectangular components referred to the planetary ephemeris frame and the radio frame, respectively. Each of the rotation matrices R_z, R_y, and R_x starts with the current orientation of the rectangular coordinate system and rotates it about its z, y, and x axes, respectively, through the rotation angles r_z, r_y, and r_x, respectively:

$$R_z = \begin{bmatrix} \cos r_z & \sin r_z & 0 \\ -\sin r_z & \cos r_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$R_y = \begin{bmatrix} \cos r_y & 0 & -\sin r_y \\ 0 & 1 & 0 \\ \sin r_y & 0 & \cos r_y \end{bmatrix} \tag{3}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos r_x & \sin r_x \\ 0 & -\sin r_x & \cos r_x \end{bmatrix} \tag{4}$$

The derivatives of the rotation matrices with respect to the rotation angles are given by:

$$\frac{dR_z}{dr_z} = \begin{bmatrix} -\sin r_z & \cos r_z & 0 \\ -\cos r_z & -\sin r_z & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$\frac{dR_y}{dr_y} = \begin{bmatrix} -\sin r_y & 0 & -\cos r_y \\ 0 & 0 & 0 \\ \cos r_y & 0 & -\sin r_y \end{bmatrix} \tag{6}$$

$$\frac{dR_x}{dr_x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin r_x & \cos r_x \\ 0 & -\cos r_x & -\sin r_x \end{bmatrix} \tag{7}$$

It should be evident in the next section why the rotation matrix $R_x R_y R_z$ was specified in Eq. (1) to rotate from the planetary ephemeris frame to the radio frame. The selection of the order of multiplication of the three matrices in Eq. (1) was arbitrary. However, once selected, all parties concerned must use the specific order specified in Eq. (1).

Table 4 on p. 22 of Ref. 1 gives the values of r_x, r_y, and r_z (labelled as A_x, A_y, and A_z) which are used to rotate from the planetary ephemeris frames for DE200 and DE234 to the radio frame. Note that the ODP should input and solve for the rotations r_x, r_y, and r_z in radians.

3. **Modified Earth-Fixed to Space-Fixed Transformation Matrix T_E**
The transformation from geocentric earth-fixed coordinates to the corresponding space-fixed coordinates referred to the radio frame is given by Eq. (17) of Ref. 2.

$$\mathbf{r}_{RF} = T_E \mathbf{r}_b \tag{8}$$

where $\mathbf{r}_b$ is the geocentric earth-fixed position vector with rectangular components referred to the true pole, equator, and prime meridian of date. The 3x3 matrix T_E is the earth-fixed to space-fixed transformation matrix which is computed from Eq. (18) of Ref. 2 as described in Section IV of Ref. 2:

$$T_E = (BNA)^T \tag{9}$$

where A is the precession matrix, N is the nutation matrix, and B is a rotation through true sidereal time θ, which is the sum of mean sidereal time θ_M and a nutation term.

The rotation matrix T_E is calculated in Rick Sunseri's Rotations Library. There is an alternative version of T_E which can be computed in this library which contains the additional rotation matrix

$$\begin{bmatrix} 1 & 0 & X \\ 0 & 1 & -Y \\ -X & Y & 1 \end{bmatrix}$$

to the left of BNA inside the parentheses in (9). The quantities X and Y are the angular coordinates of the earth's true pole of date relative to the mean pole of 1903.0. The X coordinate is measured south along the 0° meridian of 1903.0 and the Y coordinate is measured south along the 90°W meridian of 1903.0. Inclusion of the above polar motion rotation matrix in Eq. (9) changes the definition of the earth-fixed coordinate system from one referred to the true pole, prime meridian, and equator of date to one referred to the mean pole, prime meridian, and equator of 1903.0. The equations in this section will refer to T_E given by (9). The extension of these equations to the alternative form of T_E is obvious.

Let T_E' denote the 3x3 matrix which rotates from earth-fixed coordinates referred to the true pole of date to space-fixed coordinates referred to the planetary ephemeris frame:

$$\mathbf{r}_{PEF} = T_E' \mathbf{r}_b \tag{10}$$

From Eqs. (1), (8), and (9),

$$\mathbf{r}_{PEF} = (R_x R_y R_z)^T (BNA)^T \mathbf{r}_b = (BNAR_x R_y R_z)^T \mathbf{r}_b \tag{11}$$

Hence, from (10) and (11),

$$T'_E = (BNAR_x R_y R_z)^T \tag{12}$$

The matrices B, N and A are computed as previously except that the nutation angles $\Delta\psi$ and $\Delta\epsilon$ used in computing the nutation matrix N and the nutation term of sidereal time θ which is used in calculating the rotation matrix B are replaced with $\Delta\psi + \delta\psi$ and $\Delta\epsilon + \delta\epsilon$, respectively, as described previously in Section 1. The frame tie rotation matrices are computed from Eqs. (2) - (4) using the values of the solve-for frame tie rotation angles r_z, r_y, and r_x obtained from the GIN file.

The derivative of T'_E with respect to coordinate time ET (in the solar system barycentric or geocentric relativistic frame of reference) is given by:

$$\dot{T}'_E = \left[(\dot{B}NA + B\dot{N}A + BN\dot{A})R_x R_y R_z\right]^T \tag{13}$$

The matrices $\dot{B}$ and $\dot{N}$ contain terms linear in $(\Delta\psi)^{\cdot}$ and $(\Delta\epsilon)^{\cdot}$. These angular rates need to be supplemented with the time derivatives $(\delta\psi)^{\cdot}$ and $(\delta\epsilon)^{\cdot}$ of the nutation corrections, as described previously in Section 1. The second time derivative of T'_E can be evaluated with the usual approximation:

$$\ddot{T}'_E \approx (\ddot{B}NAR_x R_y R_z)^T \tag{14}$$

which can be computed by calculating

$$\ddot{T}'_E = -T'_E \dot{\theta}^2 \tag{15}$$

where the sidereal rate $\dot{\theta}$ is available from the calculation of $\dot{B}$, and then setting column three of $\ddot{T}'_E$ to zero.

From (12), the partial derivatives of T'_E with respect to the frame tie rotation angles are given by

$$\frac{\partial T'_E}{\partial r_x} = (BNA\frac{dR_x}{dr_x}R_y R_z)^T \tag{16}$$

$$\frac{\partial T'_E}{\partial r_y} = (BNAR_x\frac{dR_y}{dr_y}R_z)^T \tag{17}$$

$$\frac{\partial T'_E}{\partial r_z} = (BNAR_x R_y\frac{dR_z}{dr_z})^T \tag{18}$$

where the derivatives of R_x, R_y, and R_z with respect to r_x, r_y, and r_z are given by Eqs. (7), (6), and (5). The partial derivatives of $\dot{T}'_E$ and $\ddot{T}'_E$ with respect to these parameters are not used.

The rotation matrices $T_E, \dot{T}_E$, and $\ddot{T}_E$ should be replaced with the matrices $T'_E, \dot{T}'_E$, and $\ddot{T}'_E$ given by Eqs. (12) - (14). Note that there are two forms for each of these matrices (with and without the polar motion rotation matrix given above and its time derivative). The form used will be the one requested by the user program. Link REGRES will use the partial derivatives of T'_E with respect to the frame tie rotation angles given by Eqs. (16) - (18) (without the polar motion rotation matrix).

Certain subordinate matrices of T_E and $\dot{T}_E$ are requested by a number of programs. The nutation-precession matrix NA of T_E should be replaced by the matrix $(NA)'$ of T'_E:

$$(NA)' = NAR_xR_yR_z \tag{19}$$

The matrix $(NA)^\cdot$ should be replaced by $[(NA)']^\cdot$ given by

$$[(NA)']^\cdot = (\dot{N}A + N\dot{A})R_xR_yR_z \tag{20}$$

Sidereal time θ used in computing the rotation matrix B and the sidereal rate $\dot{\theta}$ used in computing $\dot{B}$ are used by a number of programs. The angle θ and its derivative $\dot{\theta}$ with respect to coordinate time ET are computed as before, but with $\Delta\psi$ and $\Delta\epsilon$ replaced with $(\Delta\psi+\delta\psi)$ and $(\Delta\epsilon+\delta\epsilon)$, as discussed previously. Also, in $\dot{\theta}$, $(\Delta\psi)^\cdot$ and $(\Delta\epsilon)^\cdot$ need to be supplemented with $(\delta\psi)^\cdot$ and $(\delta\epsilon)^\cdot$.

4. **Quasar Unit Vectors**

The unit vector to a quasar is given by Eq. (50) of Ref. 3:

$$\mathbf{L}_Q = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} \tag{21}$$

where α and δ are the right ascension and declination of the quasar, obtained from the GIN file. These angles and $\mathbf{L}_Q$ are referred to the radio frame. For each quasar on the GIN file, form $\mathbf{L}_Q$ from (21) and rotate it from the radio frame to the planetary ephemeris frame using

$$\mathbf{L}'_Q = (R_xR_yR_z)^T\mathbf{L}_Q \tag{22}$$

which was obtained from Eq. (1). In order to calculate partial derivatives of IWQ and INQ data types with respect to α and δ, REGRES will need

$$\frac{\partial \mathbf{L}'_Q}{\partial \alpha, \delta} = (R_xR_yR_z)^T\frac{\partial \mathbf{L}_Q}{\partial \alpha, \delta} \tag{23}$$

where $\partial\mathbf{L}_Q/\partial\alpha$ and $\partial\mathbf{L}_Q/\partial\delta$ are given by Eqs. (51) and (52) of Ref. 3.

It will be seen in Section IV.F that we do not need to calculate the partial derivatives of $\mathbf{L}'_Q$ with respect to the frame tie rotation angles.

C. Models Which Will Not be Changed Now

1. Body-Fixed to Space-Fixed Transformation Matrix T_B Used For all Bodies Except the Earth

The body-fixed to space-fixed transformation matrix $T_{B\neq E}$ used for all bodies in the solar system except the earth, its first and second time derivatives $\dot{T}_B$ and $\ddot{T}_B$, and partial derivatives of T_B with respect to the parameters $\alpha_\circ$, $\dot{\alpha}_\circ$, $\delta_\circ$, $\dot{\delta}_\circ$, $W_\circ$, and $\dot{W}_\circ$ of this model are specified in Ref. 4 (Section IV), Ref. 5 (Sections III and IV), and Ref. 6. If these transformation matrices were quite accurate, it would be appropriate to refer them to the radio frame. This would require that we append the frame tie rotation matrix to $T_{B\neq E}$ as we did for T_E in Section B.3. However, since these matrices are not terribly accurate, it was decided by the Working Group on Reference Frame Standards to assume for now that $T_{B\neq E}$ transforms from body-fixed coordinates to space-fixed coordinates referred to the planetary ephemeris frame. This means that estimated values of the parameters of the model for $T_{B\neq E}$ apply only for the planetary ephemeris which was used in obtaining the estimate.

When we obtain tracking data from a Mars lander several years from now, we will have to reinstate the lander code in link REGRES. At that time, it might be appropriate to refer $T_{B\neq E}$ to the radio frame and append the frame tie rotation matrix to it.

2. Unit Vectors to Stars

Link PV uses the direction to a reference star to determine the angular orientation of certain spacecraft. The right ascension α and declination δ of the reference star are obtained from the GIN file, and the unit vector to the star can be calculated from Eq. (21). The orientation of the reference frame of the star catalog relative to the radio frame and planetary ephemeris frame is unknown. Furthermore, the direction to the reference star does not need to be known with extreme precision. Therefore, we will assume that α and δ of the reference star are referred to the planetary ephemeris frame, and the unit vector to the star, used in PV, can be computed directly from Eq. (21). So, at the present time, this calculation will be independent of the frame tie rotation.

IV. CALCULATIONS IN LINK REGRES WHICH ARE AFFECTED BY THE MODEL CHANGES

This section lists the calculations in link REGRES which are affected by the model changes listed in Section III.B.

A. The modified transformation matrices T'_E, $\dot{T}'_E$, and $\ddot{T}'_E$ for the earth given by Eqs. (12) - (14) are used along with the earth-fixed geocentric position vector $\mathbf{r}_b$ of the tracking station to calculate the geocentric space-fixed position, velocity, and acceleration vectors of the tracking station in the planetary ephemeris frame using Eqs. (17) of Ref. 2. For spacecraft data types, these calculations are performed at the reception time $t_3(ET)$ at the receiving station on earth and at the transmission time $t_1(ET)$ at the transmitting station on earth (for round trip data types). For quasar VLBI data types, these calculations are performed at the reception times $t_1(ET)$ and $t_2(ET)$ at receiving stations 1 and 2 on earth.

   The geocentric space-fixed position vector of the tracking station in the planetary ephemeris frame, calculated as described above, is referred to the geocentric relativistic space-time frame of reference. This position vector can be used directly if the ODP is operating in this relativistic frame of reference. However, if the ODP is operating in the solar system barycentric relativistic space-time frame of reference, then the geocentric space-fixed position vector of the tracking station (in the PEF) must be transformed from the geocentric to the solar system barycentric relativistic frame using Eq. (17) of Ref. 8. This calculation is described in Ref. 2, Section VII, Item 3.

B. REGRES must calculate the partial derivatives of the geocentric space-fixed position vector of the tracking station (in the PEF) with respect to the frame tie rotation angles. From Eq. (10),

$$\frac{\partial \mathbf{r}_{PEF}}{\partial r_x} = \frac{\partial T'_E}{\partial r_x}\mathbf{r}_b \qquad x \rightarrow y, z \tag{24}$$

   where $\partial T'_E/\partial r_x, r_y$, and r_z are computed from Eqs. (16) - (18), using Eqs. (7), (6), and (5). For spacecraft data types, these calculations are performed at $t_3(ET)$ at the receiving station and at $t_1(ET)$ at the transmitting station (for round trip data types). These terms will contribute to partial derivatives of the computed spacecraft observable with respect to r_x, r_y, and r_z.

   For quasar VLBI data types, the partial derivatives (24) for the reception times $t_1(ET)$ and $t_2(ET)$ at receiving stations 1 and 2 on earth should be set to zero. This will be explained in Section F.

C. The partial derivatives of the geocentric space-fixed position vector of the tracking station on earth with respect to the parameters which affect its earth-fixed position vector are currently computed from Eq. (22) of Ref. 7. In this equation, replace T_E given by Eq. (9) with T'_E given in Eq. (12).

D. The partial derivative of the geocentric space-fixed position vector of the tracking station with respect to $UT1$ is given by Eqs. (32) and (34) of Ref. 7. This equation is evaluated at the reception time t_3 and transmission time t_1 for spacecraft data types, and at the reception times t_1 and t_2 for quasar VLBI data types. In evaluating Eq. (34) of Ref. 7, the matrix NA must be replaced by $(NA)'$ given by Eq. (19). Also, after the changes specified in Section III.B are implemented, the nutation corrections $\delta\psi$ and $\delta\epsilon$ will be used in calculating the nutation matrix N of $(NA)'$ and sidereal time θ appearing explicitly in Eq. (34) of Ref. 7.

E. In the calculation of auxiliary angles, replace T_E given by Eq. (9) with T'_E given by Eq. (12). In the calculation of the computed values of angular observables, replace the nutation-precession matrix NA with $(NA)'$ given by Eq. (19). Also, sidereal time θ, which is used liberally in these calculations, will contain the nutation corrections $\delta\psi$ and $\delta\epsilon$.

F. Computed values of quasar VLBI data types (INQ and IWQ) are calculated from Eqs. (14), (16) - (20) and (30) - (32) of Ref. 3. These equations are evaluated with geocentric space-fixed position vectors in the planetary ephemeris frame of receiving stations 1 and 2 at the reception times $t_1(ET)$ and $t_2(ET)$, obtained as described above in Section A. Also, the unit vector $\mathbf{L}_Q$ to the quasar in these equations is replaced with $\mathbf{L}'_Q$ given by Eq. (22). The net effect of the frame tie rotation on the computed value of the quasar delay τ given by Eq. (32) of Ref. 3 is very small. For a frame tie rotation of 10^{-7} rad, the computed delay τ will change by 0.06 mm or less, which is negligible.

The partial derivatives of the space-fixed position vectors of receiving stations 1 and 2 with respect to the frame tie rotation angles could be calculated from Eq. (24) and substituted into Eq. (37) of Ref. 3. This would give significant partials of the quasar delay τ with respect to the frame tie rotation angles. However, partial derivatives of $\mathbf{L}'_Q$ with respect to these angles could be obtained by differentiating Eq. (22). Substituting these partials into Eq. (53) of Ref. 3 (instead of $\partial \mathbf{L}_Q/\partial\alpha, \delta$) would give additional partials of τ with respect to the frame tie rotation angles. These two terms of the partials of the quasar delay τ with respect to the frame tie rotation angles are very nearly equal in magnitude and opposite in sign and the net partials are negligible.

Instead of calculating the above terms, which add up to nothing, let's obtain the same result by the following procedure. First, don't calculate the partials of the quasar delay τ with respect to the frame tie rotation angles due to the variation of $\mathbf{L}'_Q$ in the PEF. Second, set the partial derivatives of the geocentric space-fixed position vectors of receiving stations 1 and 2 (in the PEF) with respect to the frame tie rotation angles to zero in Eq. (37) of Ref. 3 (instead of computing them from Eq. (24) as described in Section B above). This will give partial derivatives of the computed values of wideband (IWQ) and narrowband (INQ) quasar observables with respect to the frame tie rotation angles which are zero. The partial derivatives for all other parameters are computed from the formulation of Ref. 3, Section II.E. The effects of the frame tie rotation angles on the computed values of these partial derivatives are very small. In Eq. (53) of Ref. 3, $\partial \mathbf{L}_Q / \partial \alpha, \delta$ must be replaced with $\partial \mathbf{L}'_Q / \partial \alpha, \delta$ given by Eq. (23).

V. CALCULATIONS IN LINK PV WHICH ARE AFFECTED BY THE MODEL CHANGES

This section lists the calculations in link PV which are affected by the model changes listed in Section III.B.

A. In calculating the acceleration of the spacecraft due to the earth's oblateness, atmospheric drag, mascons, albedo, infrared radiation, solid earth tides, etc., replace the earth-fixed to space-fixed transformation matrix T_E given by Eq. (9) with T'_E given by Eq. (12). In calculating the acceleration due to drag, $\dot{T}'_E$ given by (13) is also used. Note that link PV uses the alternative forms of T'_E and $\dot{T}'_E$ which contain the polar motion rotation matrix discussed in Section III.B.3 and its time derivative.

B. As discussed in Section II, don't calculate the partial derivatives of the spacecraft acceleration with respect to the frame tie rotation angles r_x, r_y and r_z. Hence, the integrated dynamic partial derivatives of the spacecraft state vector with respect to these parameters will be zero.

VI. CALCULATIONS IN OTHER LINKS WHICH ARE AFFECTED BY THE MODEL CHANGES

The matrix T'_E given by Eq. (12) and its time derivative $\dot{T}'_E$ given by Eq. (13) are used in transforming from space-fixed coordinates referred to the planetary ephemeris frame to earth-fixed coordinates. These coordinate transformations can also be performed using the alternate forms of T'_E and $\dot{T}'_E$ which contain the polar motion rotation and its time derivative. Either form of these coordinate transformations can be performed in links GIN, TWIST, STATRJ, and MAPGEN. The precession-nutation matrix $(NA)'$ given by Eq. (19) and its time derivative $[(NA)']^{\cdot}$ given by Eq. (20) can also be used by these links.

Distribution

C. H. Acton
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